Graph Theory and Social Networks - part 3

EE599: Social Network Systems

Keith M. Chugg

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USC University of Southern California

Overview

- Continuation of graph theory for social networks
  - Typical social network properties
  - Graph measures to quantify
  - Examples

References

- Easley & Kleinberg, Ch 3
  - Focus on relationship to social nets with little math
- Jackson, Ch 2-3
  - Social network focus with more formal math

Motivation

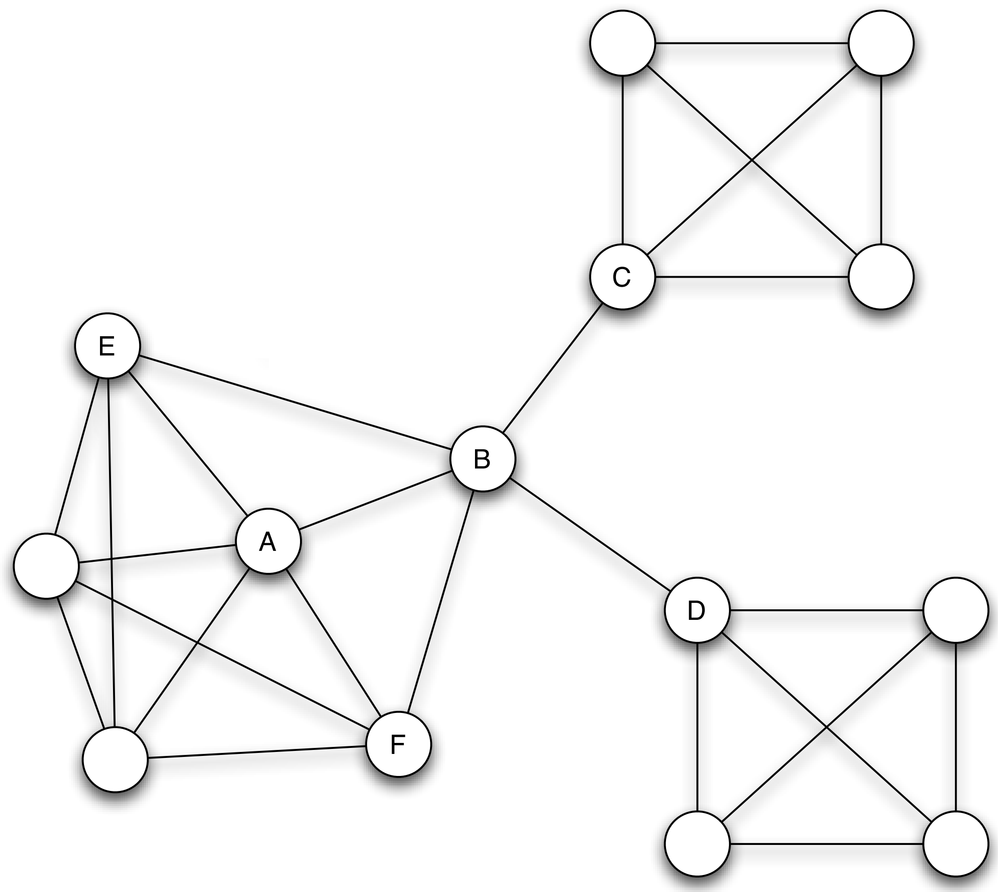


Figure 3.11: The contrast between densely-knit groups and boundary-spanning links is reflected in the different positions of nodes A and B in the underyling social network.

Easley & Kleinberg

- Not all nodes and edges are “equal”
  - A vs. B

Motivation

Our discussion thus far suggests a general view of social networks in terms of tightly-knit groups and the weak ties that link them. The analysis has focused primarily on the roles that different kinds of edges of a network play in this structure — with a few edges spanning different groups while most are surrounded by dense patterns of connections.

Easley & Kleinberg, pg. 64

- Our goal
  - Motivate this from a few social rules - simple, yet reasonable
  - Define qualitative terms and quantitative measures to capture these properties

Overview

- Triadic Closure & Cluster Coefficients
  - *Our friends usually become friends*
- Strong & Weak Ties
  - *Most people get jobs from acquaintances rather than close friends*
- Centrality and Prestige Measures
  - *Some people (or connections) are more critical than others*
    - Next time

Related Concepts & Terminology

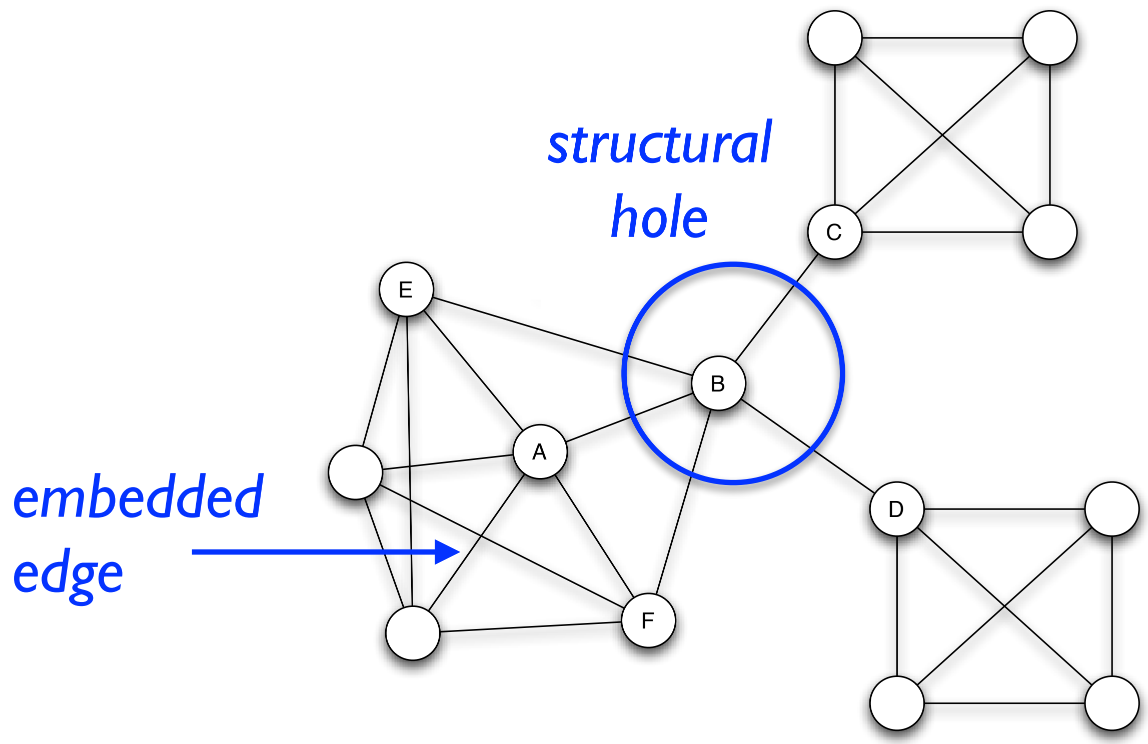


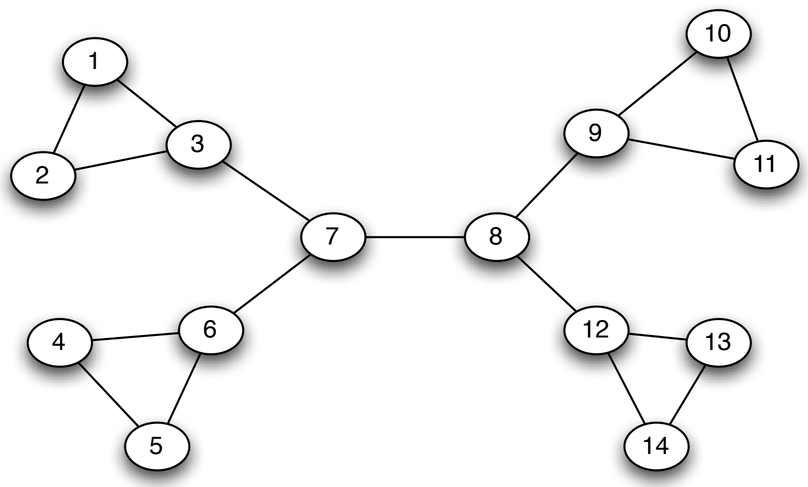
Figure 3.11: The contrast between densely-knit groups and boundary-spanning links is reflected in the different positions of nodes A and B in the underlying social network.

Easley & Kleinberg

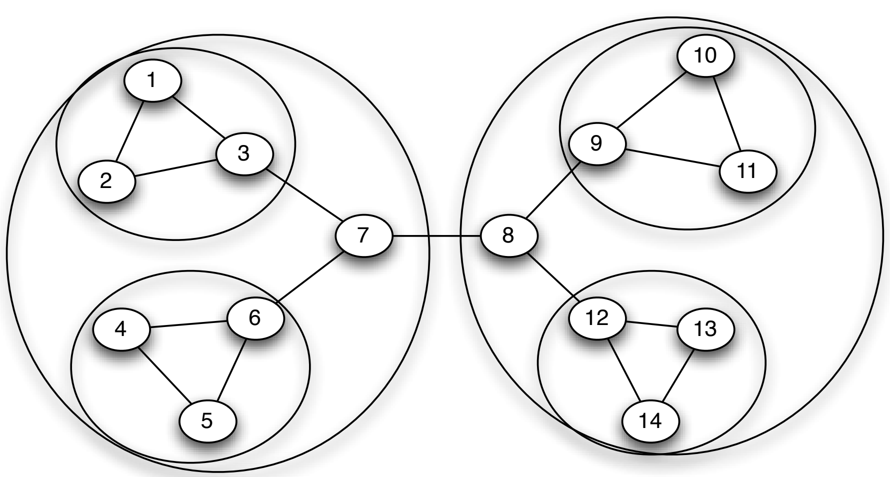
Social Capital?

- Embeddedness of edge connecting A and B
  - *number of common neighbors of A,B*
  - *A, B connected by embedded edge implies high degree of trust*
- Structural Holes are filled by nodes with access to local brides
  - *amplifies creativity*
  - *serves as a gatekeeper of information flow across sub-organizations*
  - *Can create power struggles and trust issues*

Network Partitioning



(a) *A sample network*



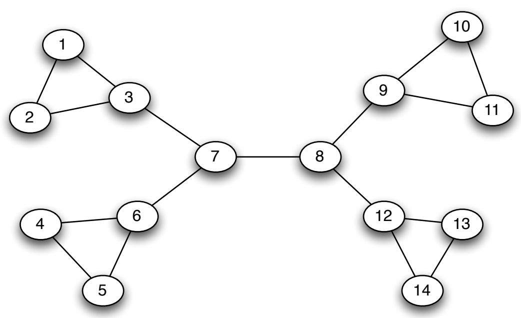
(b) *Tightly-knit regions and their nested structure*

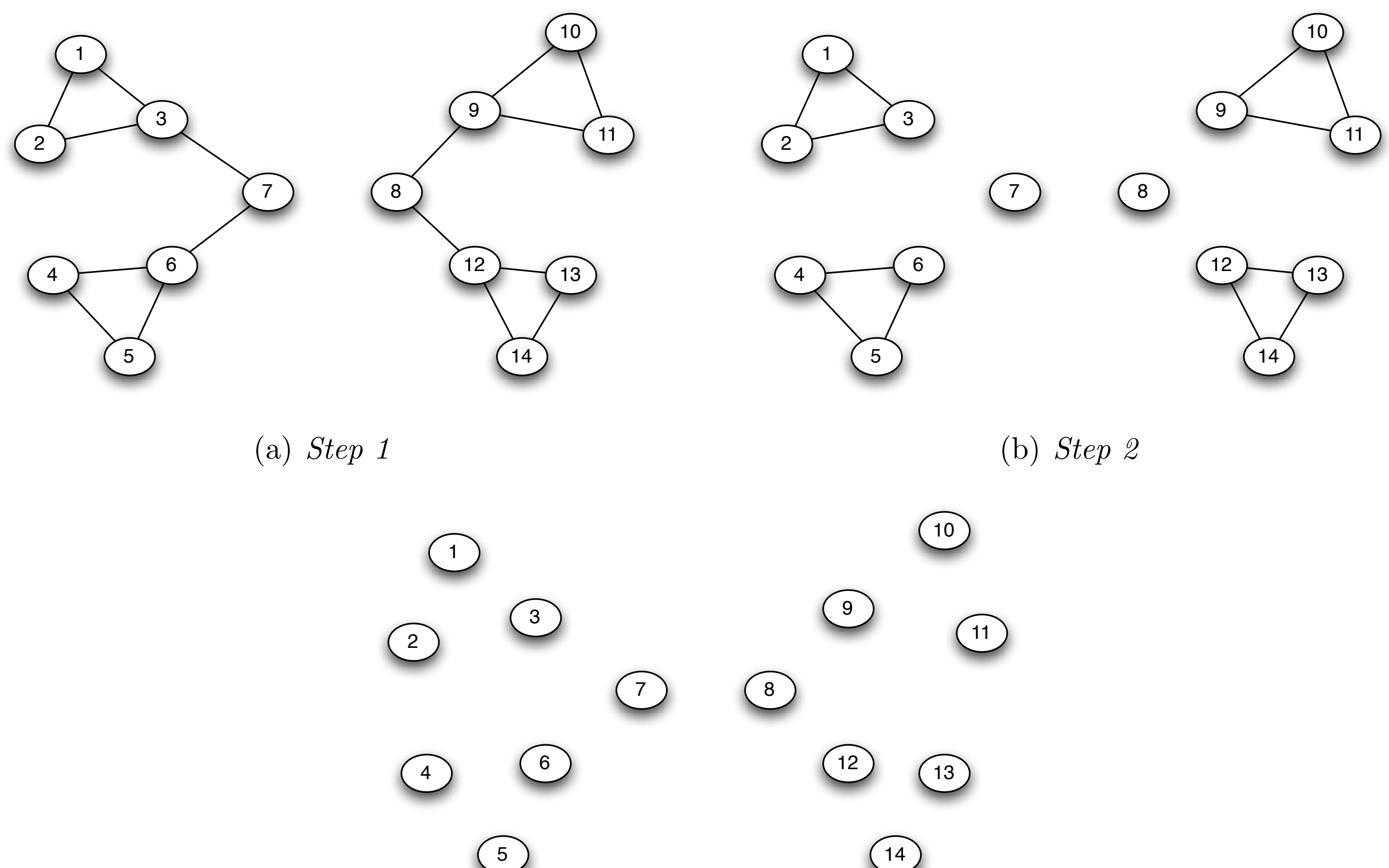
Figure 3.14: In many networks, there are tightly-knit regions that are intuitively apparent, and they can even display a *nested* structure, with smaller regions nesting inside larger ones.

Easley & Kleinberg

- How to identify tightly knight regions in a social network from the graph structure?
  - Assume we have a method of identifying the most "central" edges
  - Remove these edges to break the graph into components
  - Repeat this process on the components as they arise
- Girvan-Newman Algorithm

Network Partitioning





(a) *Step 1*

(b) *Step 2*

(c) *Step 3*

Figure 3.16: The steps of the Girvan-Newman method on the network from Figure 3.14(a).

Easley & Kleinberg

Network Partitioning

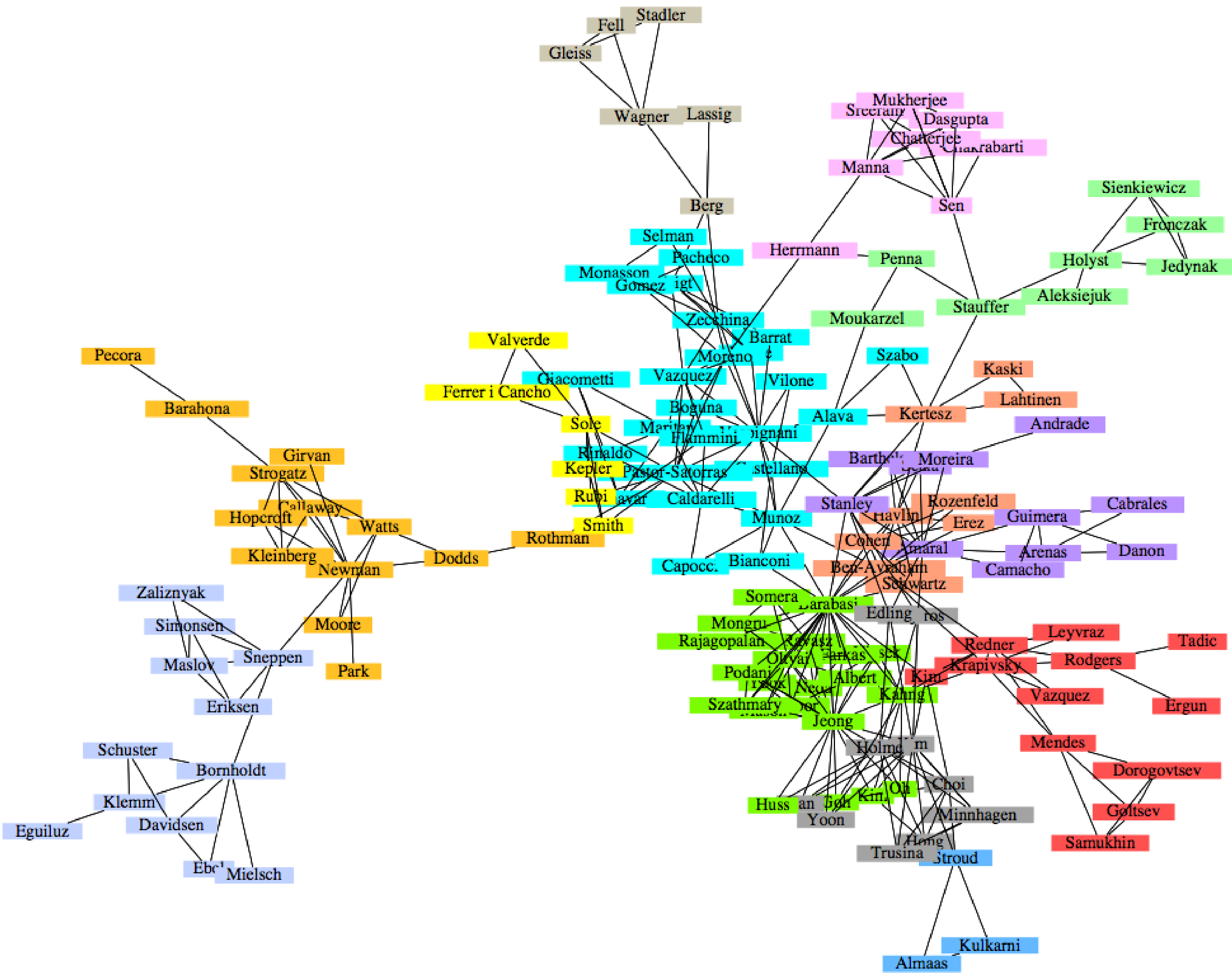


Figure 3.12: A co-authorship network of physicists and applied mathematicians working on networks [322]. Within this professional community, more tightly-knit subgroups are evident from the network structure.

Easley & Kleinberg

Network Partitioning

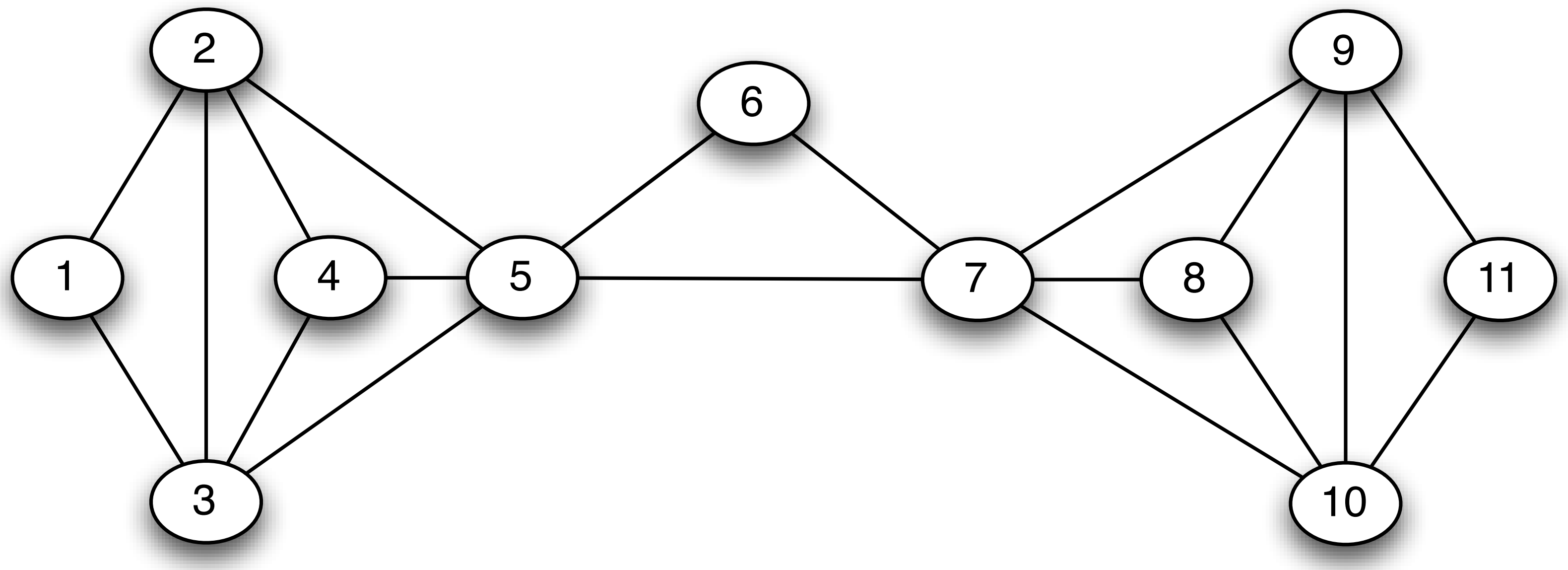


Figure 3.15: A network can display tightly-knit regions even when there are no bridges or local bridges along which to separate it.

Easley & Kleinberg

Network Partitioning

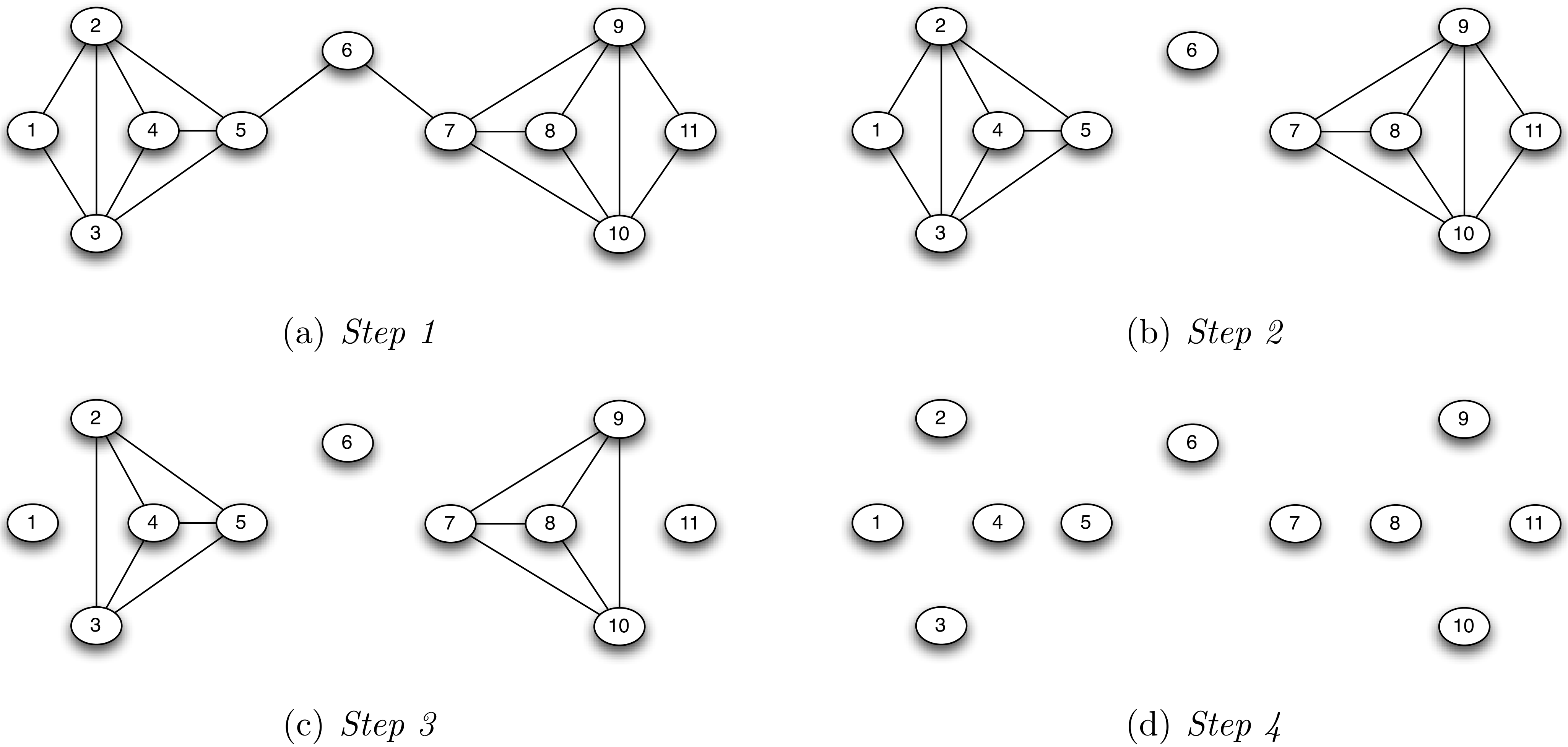


Figure 3.17: The steps of the Girvan-Newman method on the network from Figure 3.15.

Easley & Kleinberg

Centrality Measures

- How to measure the "centrality" of a given node or edge?
  - *Obvious in some cases (e.g., star graph)*
- Many measures have been proposed for measuring the importance, centrality, or prestige of a node based only on the network topology

Centrality Measures (nodes)

- Degree Centrality: $$\frac{k_i}{N-1}$$

- Closeness Centrality: $$\frac{1}{d_{\text{ave}(i)}} = (N-1)\left[\sum_{j \neq i} d(i,j)\right]^{-1}$$

- Decay Centrality: $$\sum_{j \neq i} \delta^{d(i,j)}, \quad \delta \in (0,1)$$
  - delta~0: ~degree centrality
  - delta~1: ~size of largest component containing i

Centrality Measures (nodes)

- Betweenness Centrality:
  - Fraction of shortest paths in the network that pass through node i

$$B(i) = \sum_{(j,k), i \notin \{j,k\}} \frac{P_i(j,k)}{P(j,k)} = \frac{\text{number of shortest paths between j \& k, passing through i}}{\text{number of shortest paths between j \& k}}$$

Often normalized by: $\binom{N-1}{2}$ (number of pairs excluding node i)

Centrality Measures (nodes)

- Eigen Centrality:
  - Based on the idea that importance is determined by how many important friends you have

$$\lambda p_i = \sum_{j=1}^{N} a_{ij} p_j$$

$$\mathbf{A}\mathbf{p} = \lambda \mathbf{p}$$

- Vector of node centralities is the eigenvector of the adjacency matrix with largest eigenvalue

Centrality Measures (nodes)

- Katz Prestige:
  - normalize impact of friendships by node degree

$$p_i = \sum_{j=1}^{n} a_{ij} \frac{p_j}{k_j} = \sum_{j=1}^{n} \frac{a_{ij}}{k_j} p_j = \sum_{j=1}^{n} h_{ij} p_j$$

Google's page-rank is based on an eigen centrality measure

$$\mathbf{p} = \mathbf{H}\mathbf{p}$$

- Vector of node centralities is the eigenvector of a weighted adjacency matrix with eigenvalue 1 (exists by stationary theorem of Markov Chains)

Centrality Example

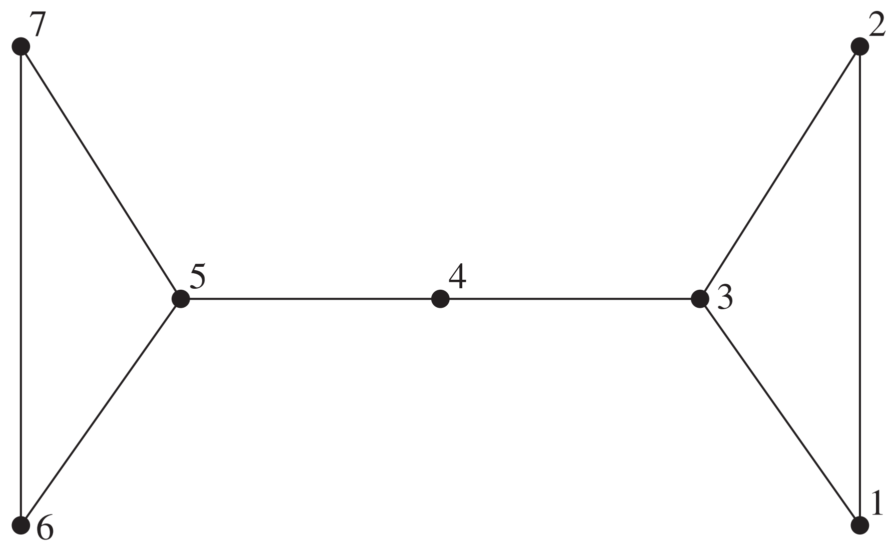


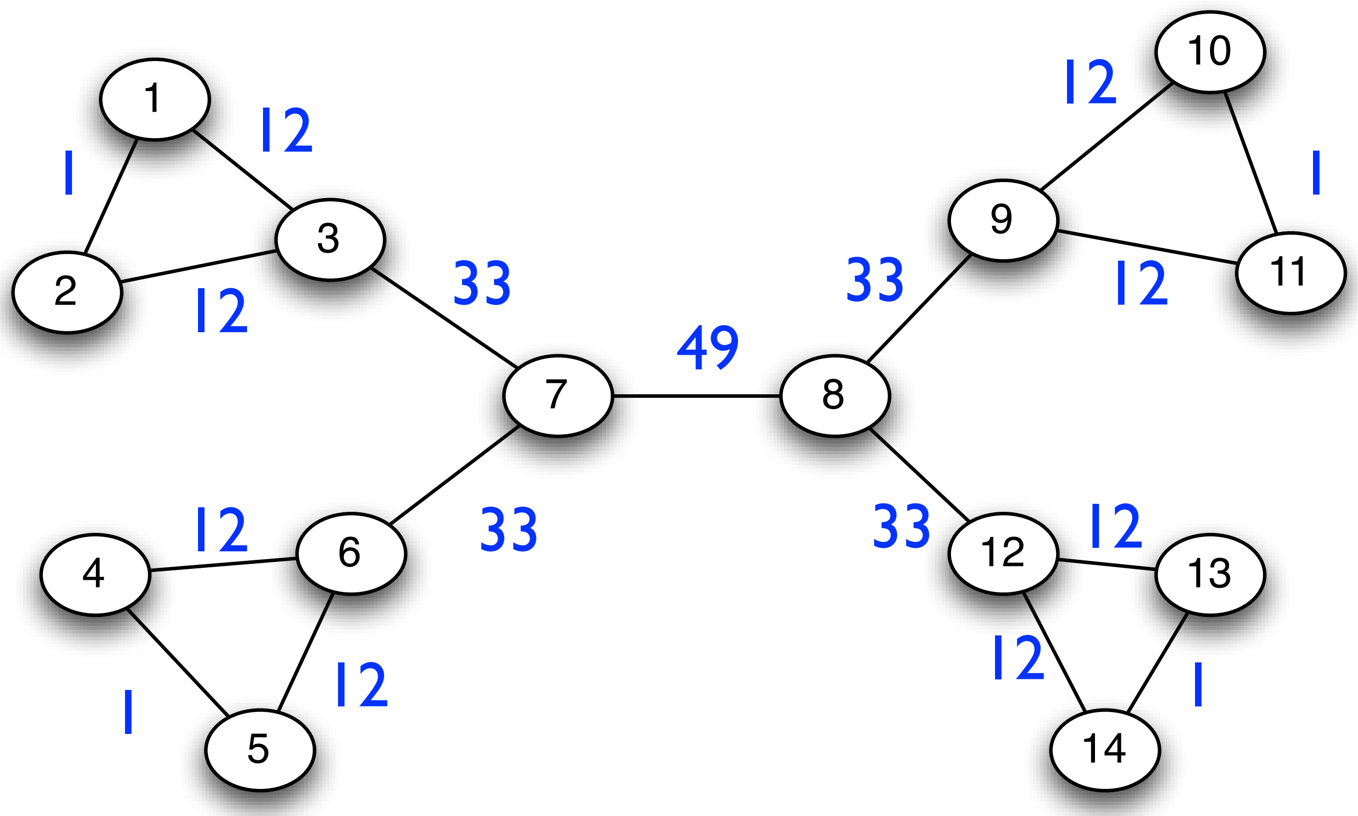
FIGURE 2.13 A central node with low degree centrality.

TABLE 2.1
Centrality comparisons for Figure 2.13

Measure of centrality	Nodes 1, 2, 6, and 7	Nodes 3 and 5	Node 4
Degree (and Katz prestige P^K)	.33	.50	.33
Closeness	.40	.55	.60
Decay centrality ($\delta = .5$)	1.5	2.0	2.0
Decay centrality ($\delta = .75$)	3.1	3.7	3.8
Decay centrality ($\delta = .25$)	.59	.84	.75
Betweenness	.0	.53	.60
Eigenvector centrality	.47	.63	.54
Katz prestige-2 P^{K2}, $a = 1/3$	3.1	4.3	3.5
Bonacich centrality $b = 1/3$, $a = 1$	9.4	13.0	11.0
Bonacich centrality $b = 1/4$, $a = 1$	4.9	6.8	5.4

Jackson

Computing Betweenness (edge)



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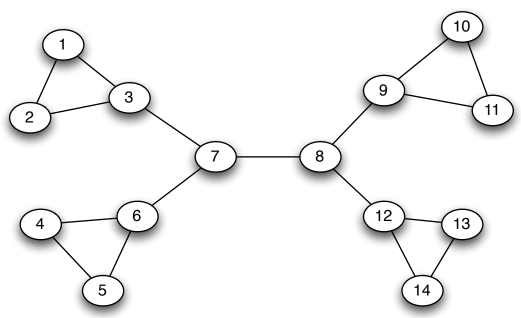
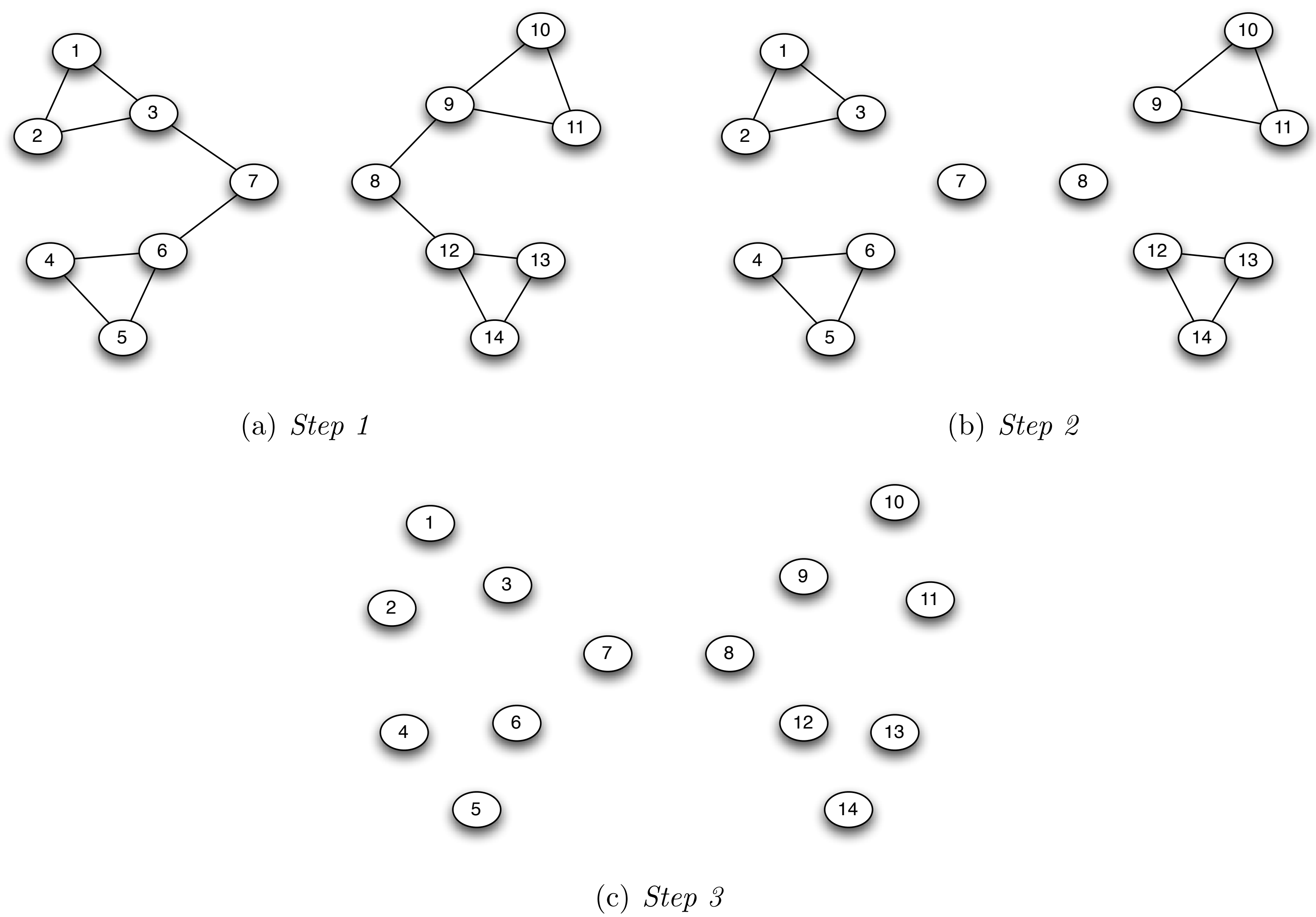
Network Partitioning





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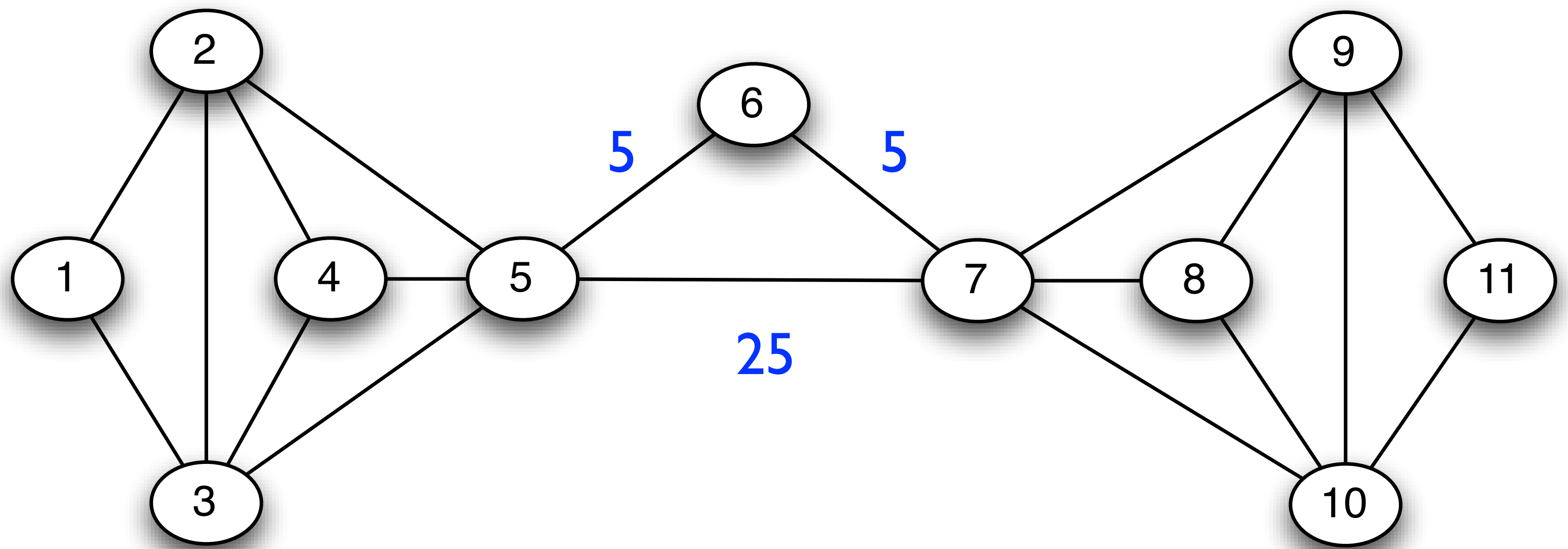

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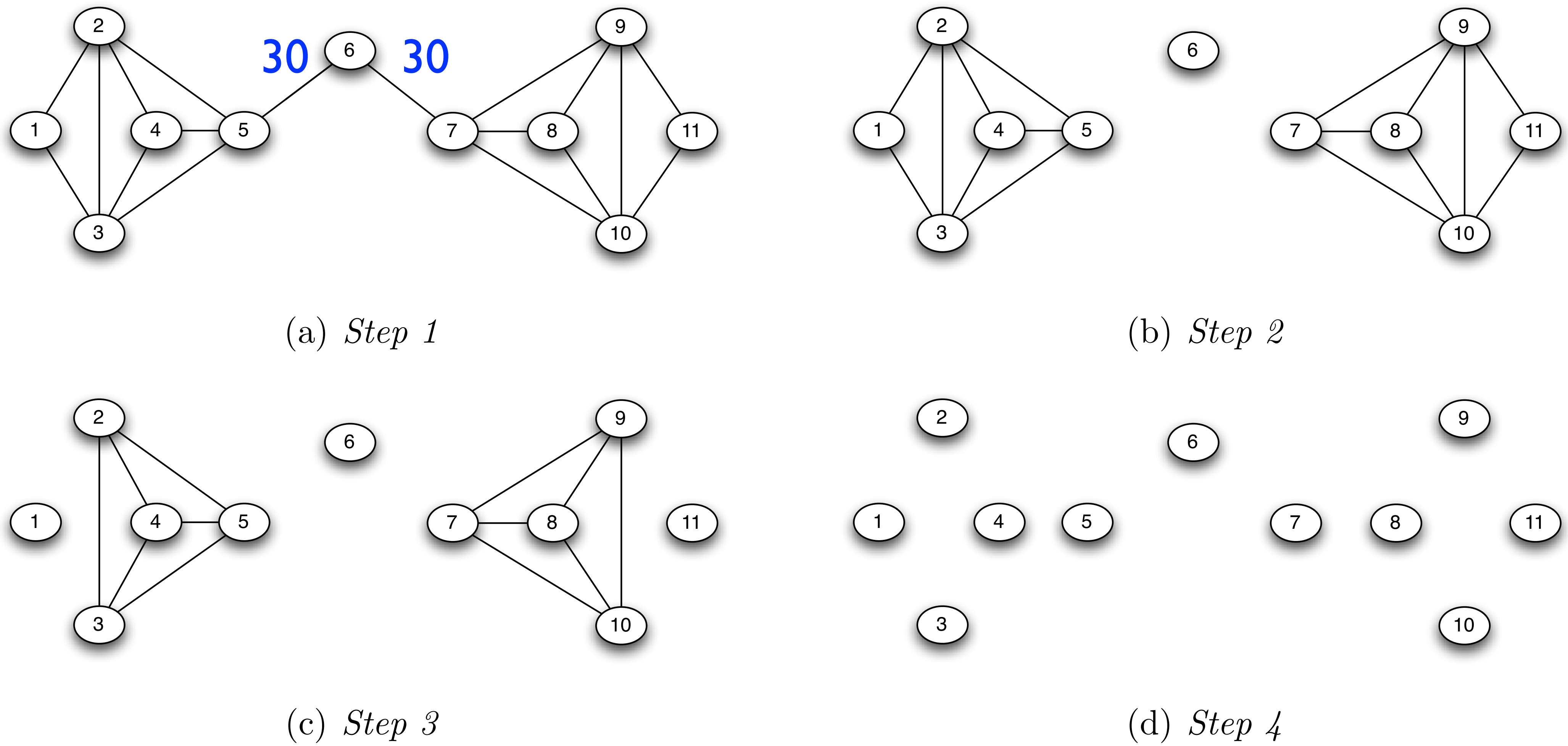
Network Partitioning



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