Iterative Message Passing on Graphs

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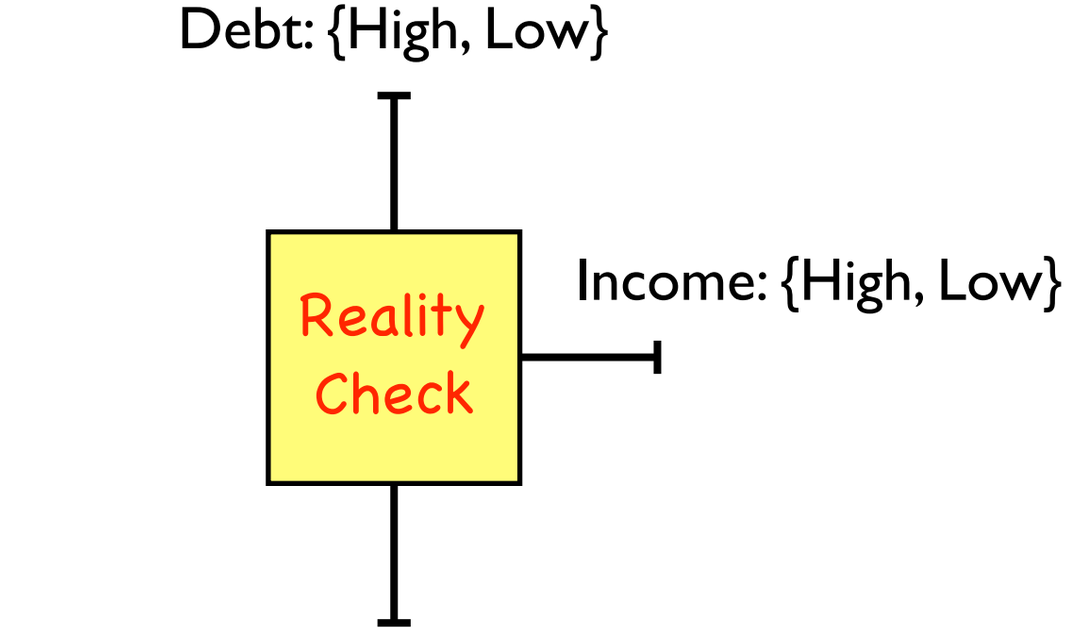
Systems: Constraints on Variables

Rating	Income	Debt
Good	Low	Low
~~Good~~	~~Low~~	~~High~~
Good	High	Low
~~Good~~	~~High~~	~~High~~
Average	Low	Low
~~Average~~	~~Low~~	~~High~~
Average	High	Low
Average	High	High
~~Bad~~	~~Low~~	~~Low~~
Bad	Low	High
~~Bad~~	~~High~~	~~Low~~
~~Bad~~	~~High~~	~~High~~

Example: Personal Finances



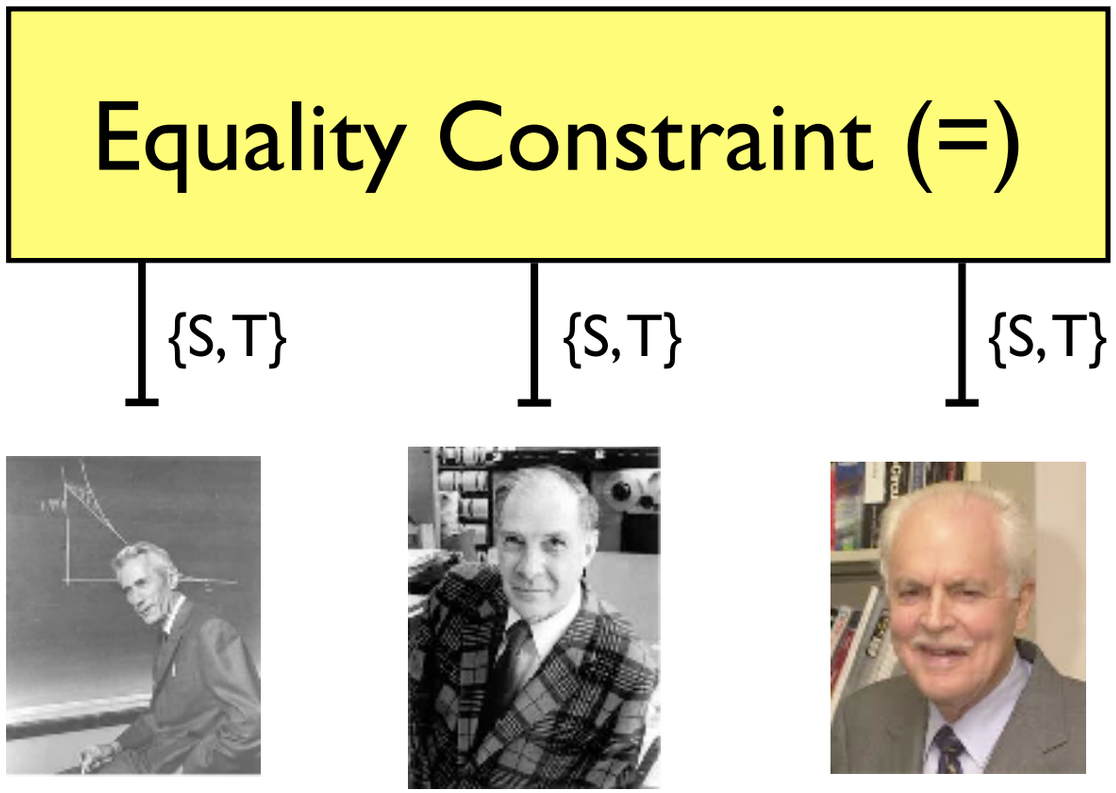
USC Lunch Group Planning

Making Decisions with Constraints

Example: Group Lunch

- Everybody goes to same place
- Choices: {Seeds, Tutor Cafe}

- S <=> '0' & T <=> '1'



Configuration	Claude	Richard	Irving
"0" (SSS)	Seeds	Seeds	Seeds
"1" (TTT)	Tutor Cafe	Tutor Cafe	Tutor Cafe

How Do We Make These Decisions?

Example: Group Lunch

Hard Decisions In

Claude	Richard	Irving
Seeds	Seeds	Tutor

Hard Decision Out

Decision


Soft Decisions In

Claude	Richard	Irving
“Either is OK, ... Seeds”	“IDK, ... I guess Seeds”	“I can’t stand Seeds... Tutor Cafe!!!!”

Hard Decision Out

Decision


Soft Decisions = degree of preference

Quantifying/Normalizing Soft Decisions

Example: Group Lunch

Soft-in:	M[S]	M[T]
Claude	4	5
Richard	8	10
Irving	20	0

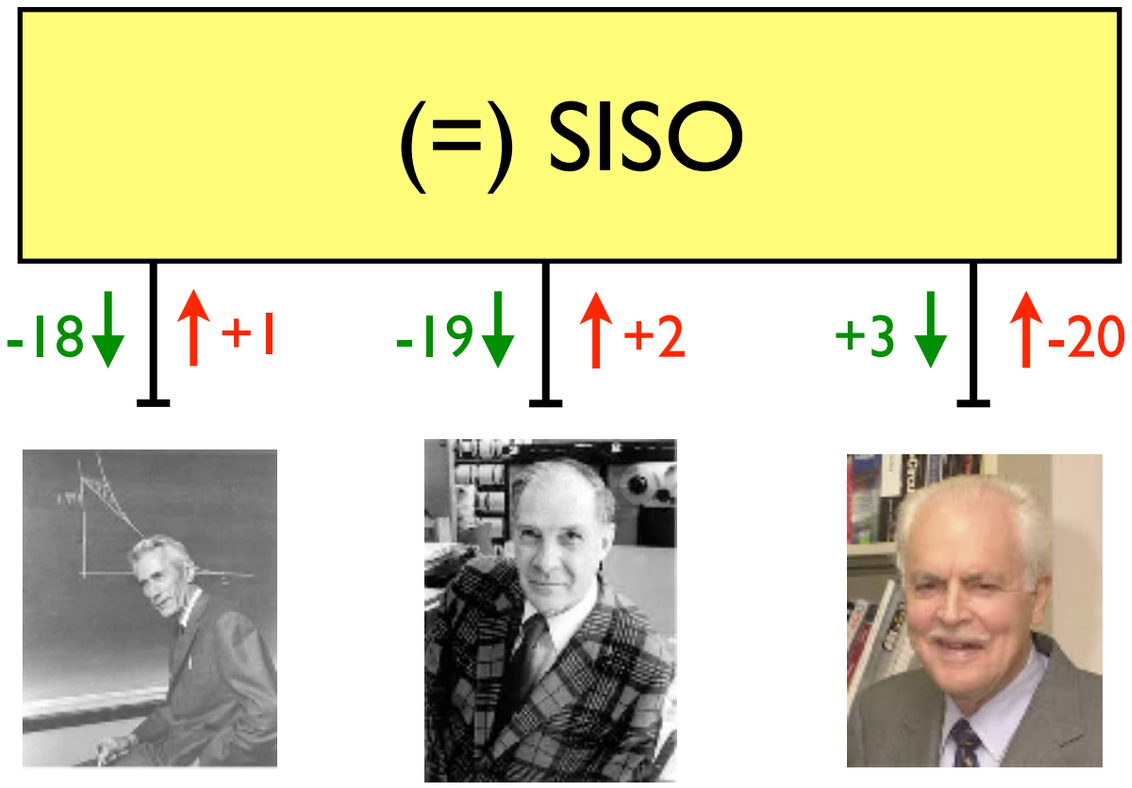
"How many 'lunch points' would somebody have to pay me?"

Soft-in:	M[S]	M[T]
Claude	0	+1
Richard	0	+2
Irving	0	-20

"How many more 'lunch points' would somebody have to pay me *to go to Tutor Cafe over Seeds?*"

It is desired to minimize the group "pain" in deciding

Soft-In/Soft-Out Lunch Decision



Config.	Claude	Richard	Irving	Config. Metric
"0" (SSS)	0	0	0	0
"1" (TTT)	+1	+2	-20	-17

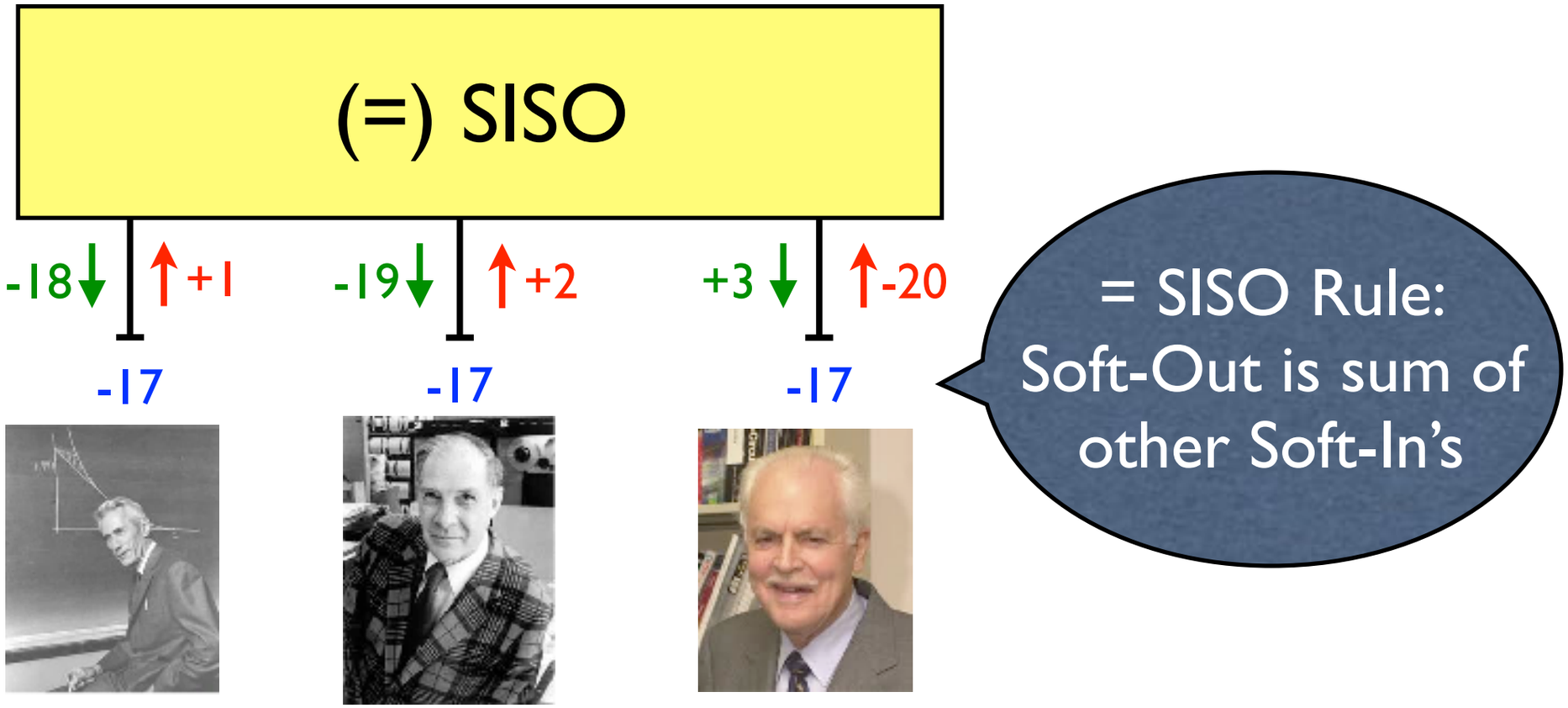
-17 (Best Config. Metric with Irving = Tutor Cafe)

0 - (Best Config. Metric with Irving = Seeds)

-(-20) - (Irving's (Normalized) Soft-In Information)

+3 Irving's (Normalized) Soft-Out Information

Lunch SISO Interpretation



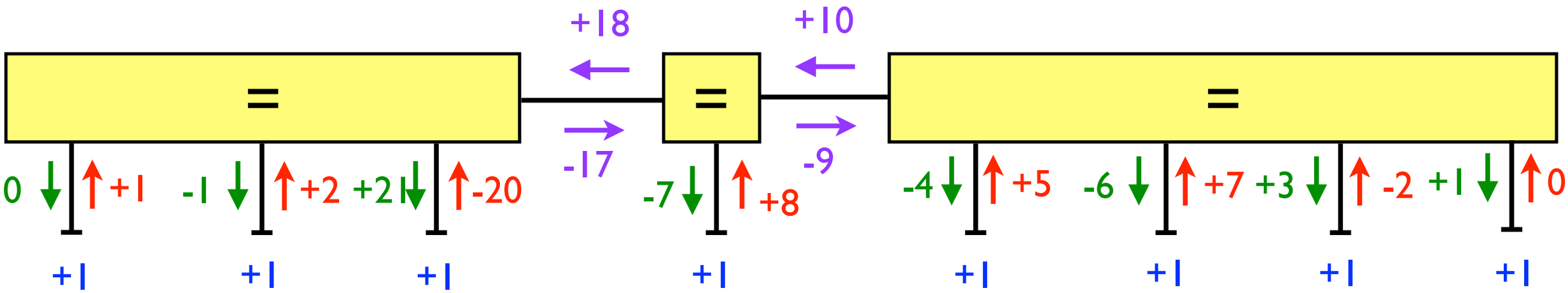
"Intrinsic Format" = Soft-In + Soft-Out

- Determines best decision
- Same for everyone -- equality constraint!

"Extrinsic Format" = Soft Out (Soft-In too)

- How much do others want to go to Tutor Cafe over Seeds?

Two Lunch Groups & Keith



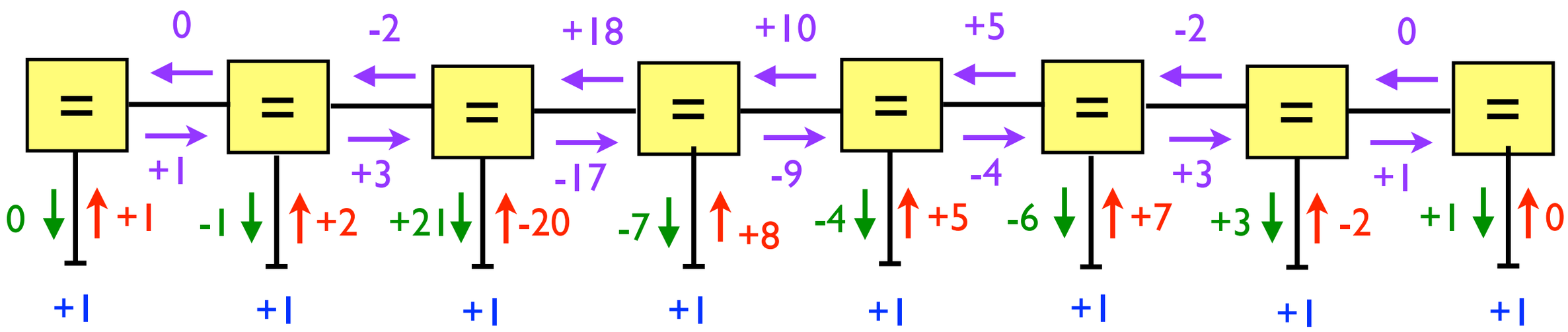




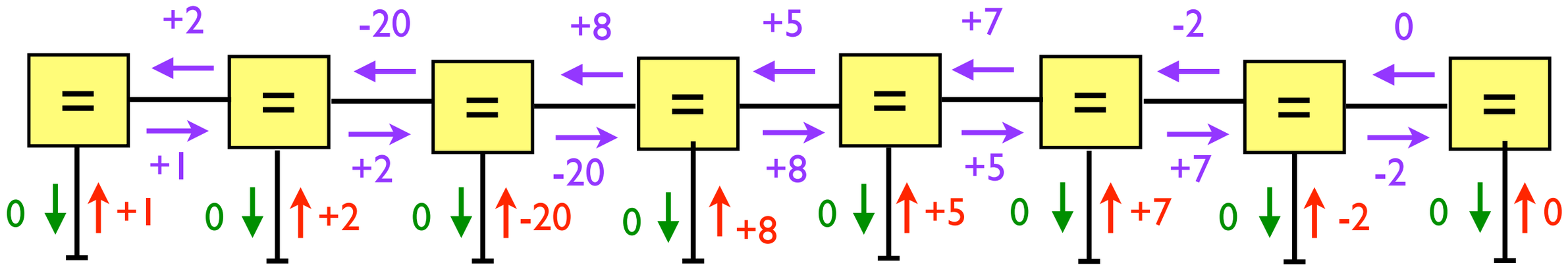



Same Decision!

Forward-Backward Algorithm

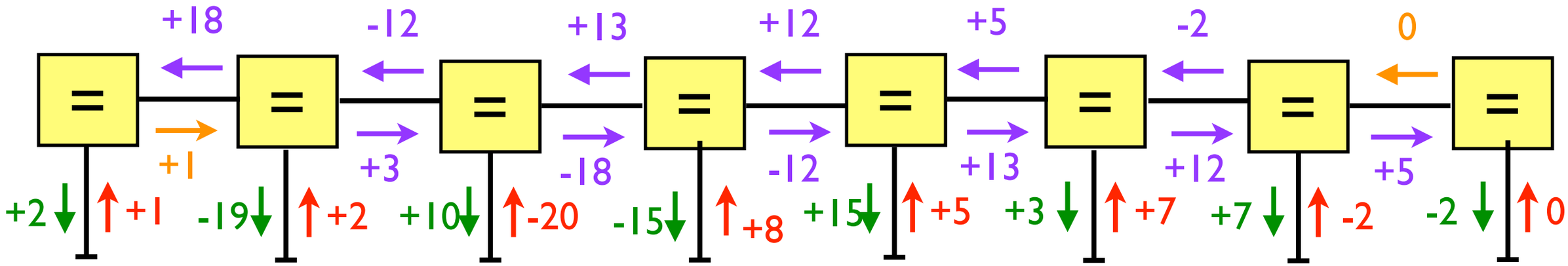


Different Activation Schedule: Flooding



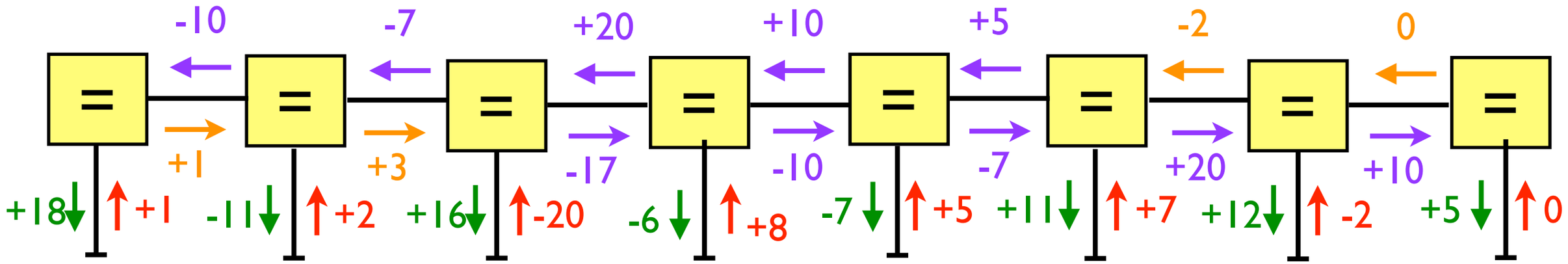
+1 +2 -20 +8 +5 +7 -2 0

Different Activation Schedule: Flooding



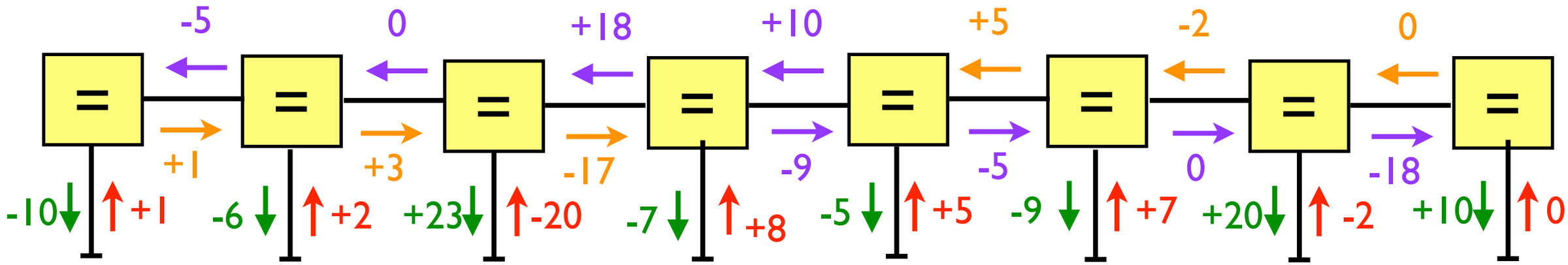
+3 -17 -10 -7 +20 +10 +5 -2

Different Activation Schedule: Flooding



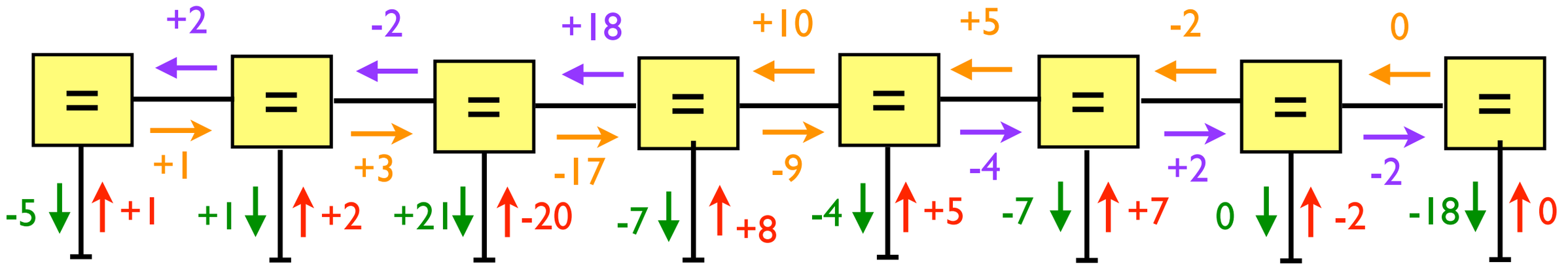
+19 -9 -4 +2 -2 +18 +10 +5

Different Activation Schedule: Flooding



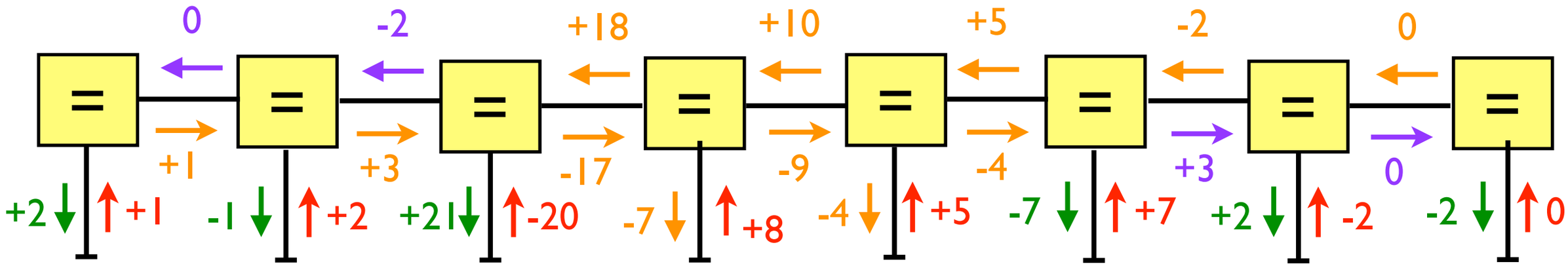
-9 -4 +3 +1 0 -2 +18 +10

Different Activation Schedule: Flooding

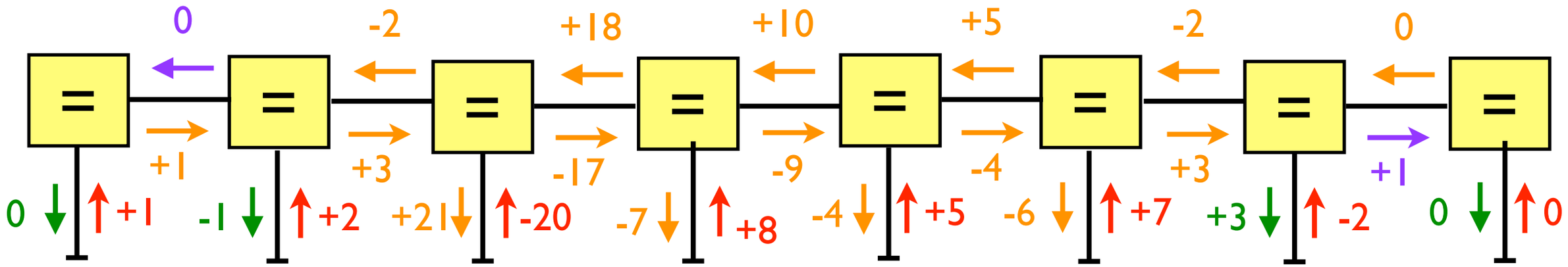


-4 +3 +1 +1 +1 +1 -2 -18

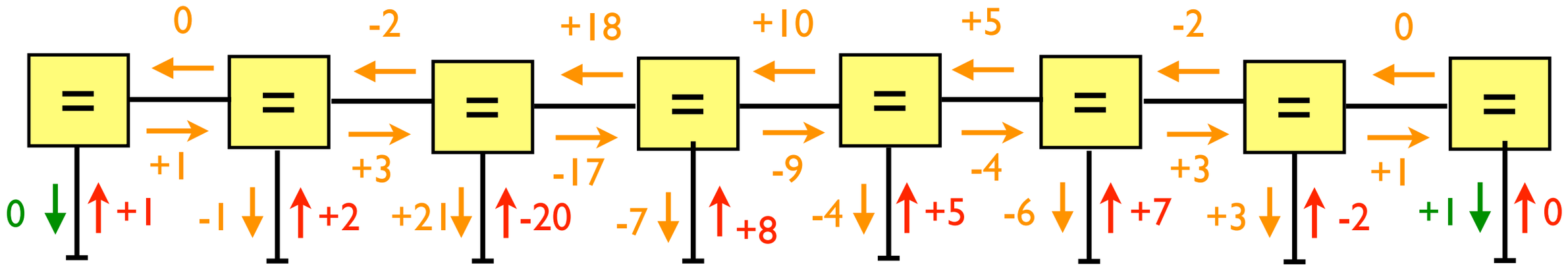
Different Activation Schedule: Flooding



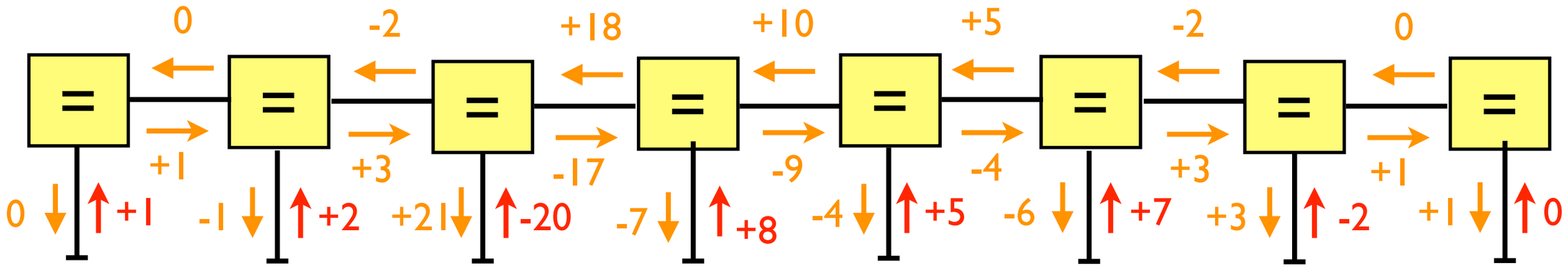
Different Activation Schedule: Flooding



Different Activation Schedule: Flooding



Different Activation Schedule: Flooding



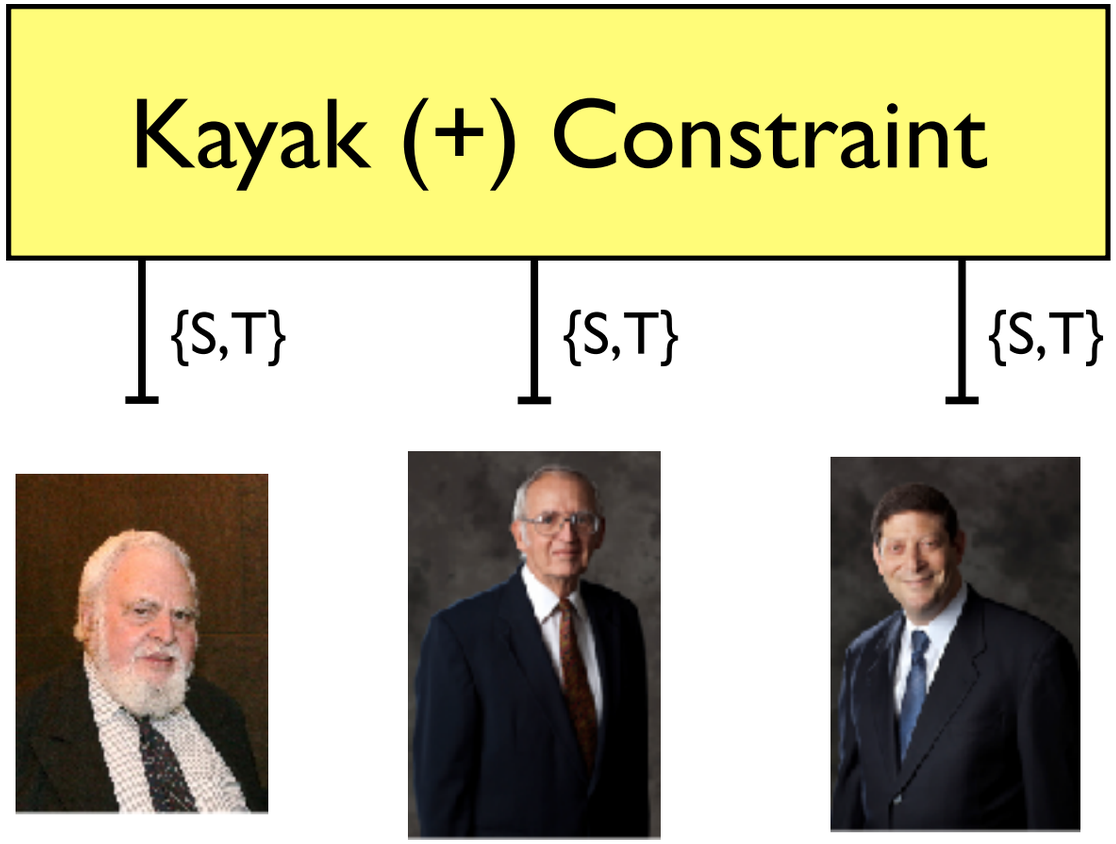
Same Result!

USC Kayak Trip Planning

Making Decisions with Constraints

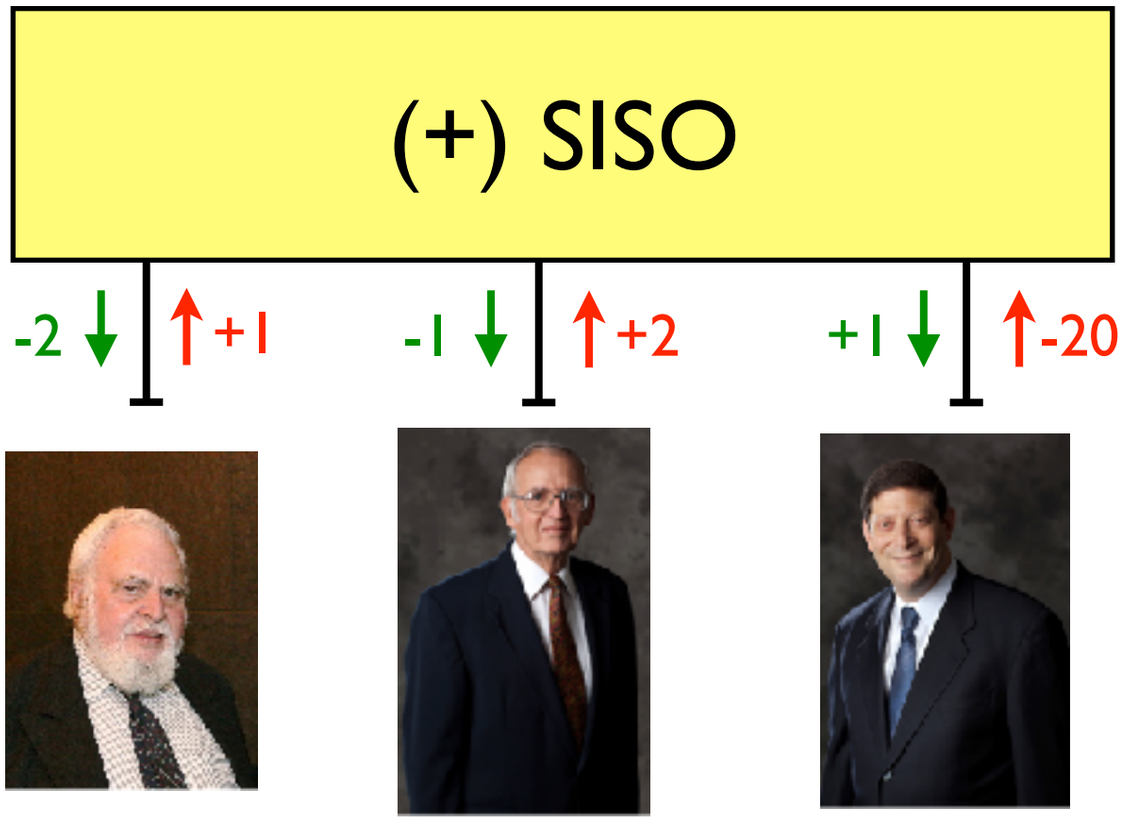
Example: Kayaking Trip

- Each person chooses: Tandem (T <=> 1) or Single (S <=> 0)
- Constraint: even number of people choose Tandem



Configuration	Sol	Bob	Alan
0	S	S	S
1	S	T	T
2	T	S	T
3	T	T	S

Soft-In/Soft-Out Kayak Decision



Configuration	Sol	Bob	Alan	Config. Metric
0 (SSS)	0	0	0	0
1 (STT)	0	+2	-20	-18
2 (TST)	+1	0	-20	-19
3 (TTS)	+1	+2	0	+3

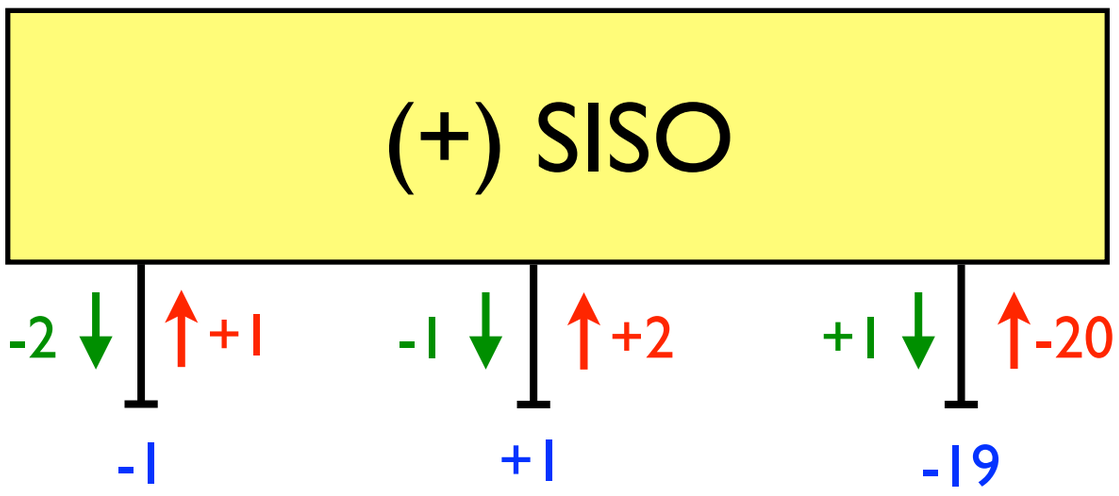
-18 (Best Config. Metric with Bob = Tandem)

-(-19) - (Best Config. Metric with Bob = Single)

-(+2) - (Bob's (Normalized) Soft-In Information)

-1 Bob's (Normalized) Soft-Out Information

Kayak SISO Interpretation



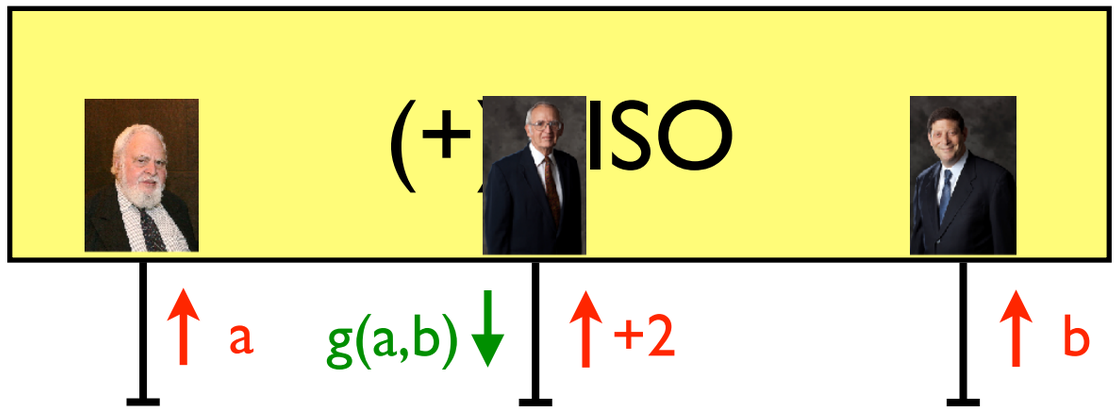
"Intrinsic Format" = Soft-In + Soft-Out

- Determines best decision
- Even number of Tandems -- + constraint!

"Extrinsic Format" = Soft Out

- Others best case cost for you going Tandem (odd number of other Tandems) normalized to their best case cost of you going Single (even number of other tandems)

Kayak SISO Interpretation

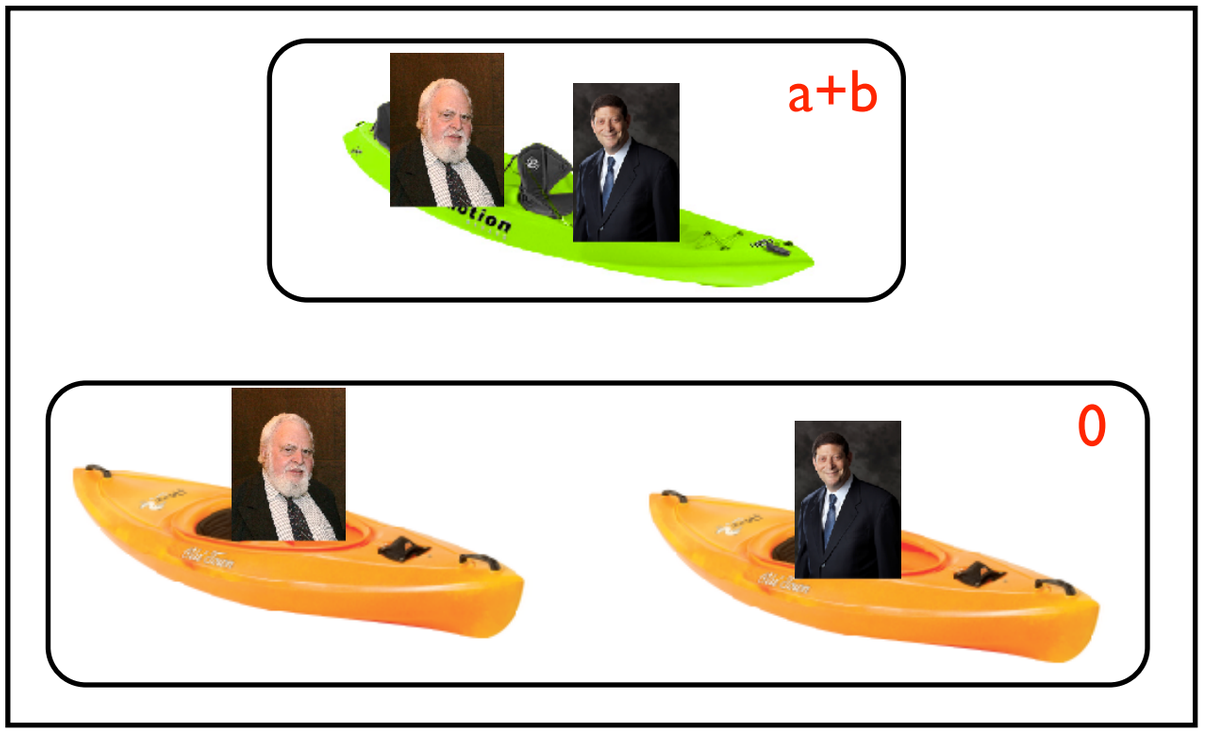


$$g(a,b) = \min(a,b) - \min(0,a+b) = \mathrm{sign}(a)*\mathrm{sign}(b)*\min\{|a|,|b|\}$$

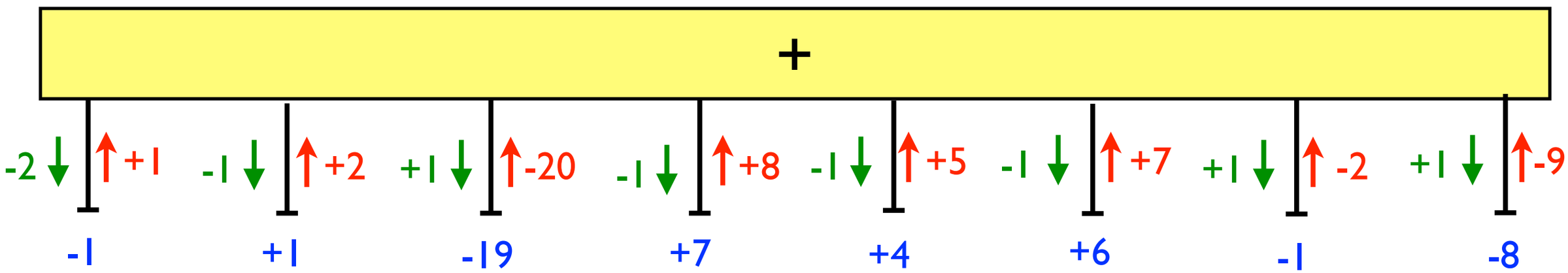
"1"

"0"

-

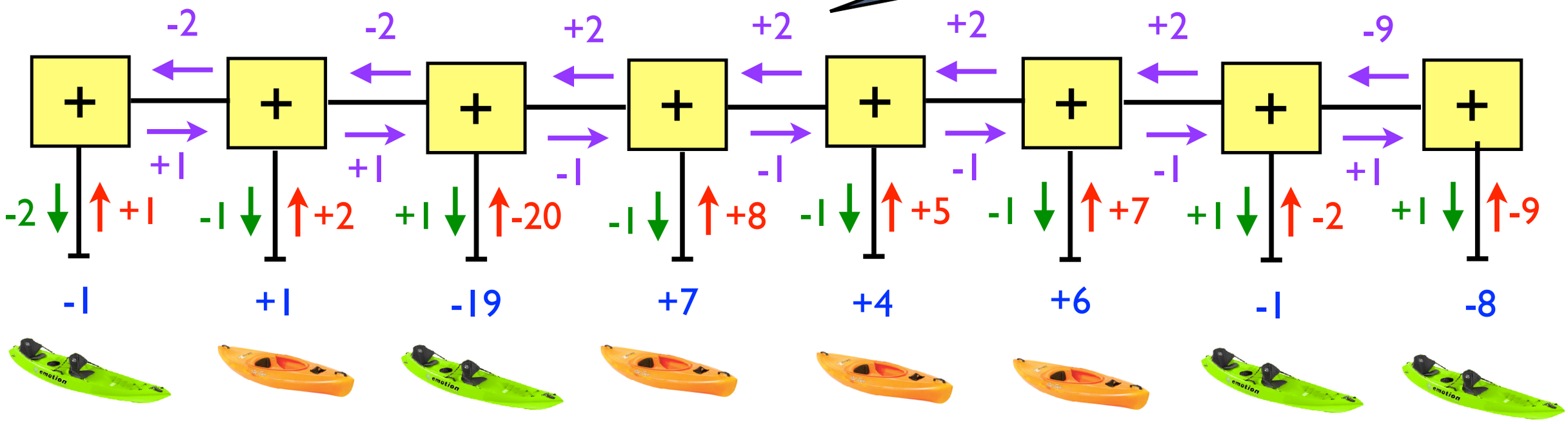


8 Person Kayak Trip



Same Decisions!

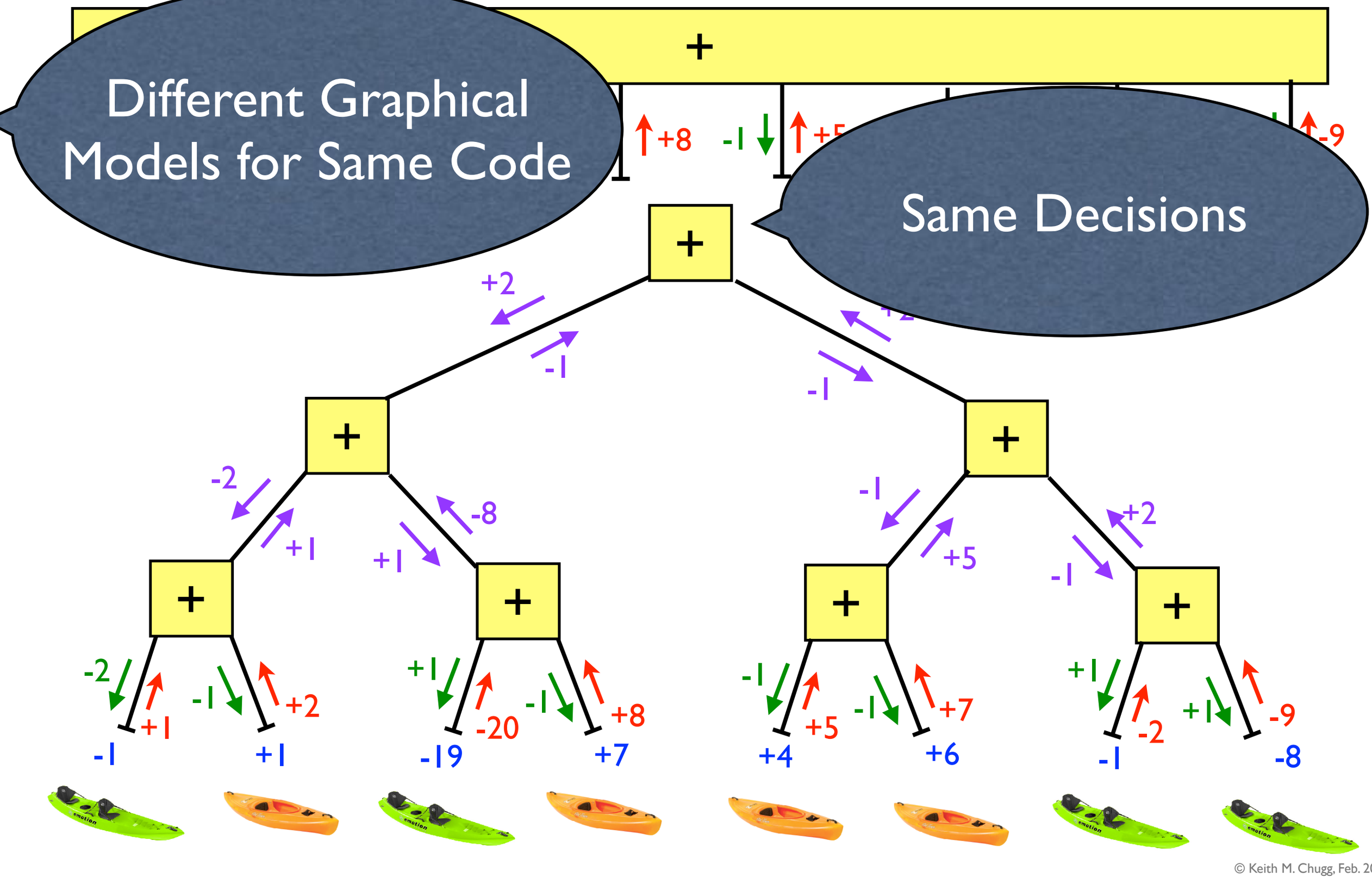
Forward-Backward Algorithm



8 Person Kayak Trip - Tree Graph

Different Graphical Models for Same Code

Same Decisions



Planning Multiple Kayak Trips

3 Kayak Trips & 6 People



-5 0 +5

-4 -1 +1 -1 +6 -1

Kayak Trip 1 Kayak Trip 2 Kayak Trip 3

+1 +3 -1 +3 -3 -1 +1 -1 +2 -1 -1 +2 +2

= = =

0 +3 0 -1 -2 +2



+3 -1 0

3 Kayak Trips & 6 People

-5

-4 ↓ ↑ -1

+ Kayak Trip 1

0

+1 ↓ ↑ -1

+ Kayak Trip 2

+5

+6 ↓ ↑ -1

+ Kayak Trip 3

up

↑ +2 ↑ +4 ↑ +2 ↑ -2 ↑ -3 ↑ +1 ↑ +1

= 0 ↓ ↑ +3

+3

= 0 ↓ ↑ -1

-1

= -2 ↓ ↑ +2

0

2

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

0

+1 -1

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

down

-2 -1 -2 +1 +1 -1 -3

=

0 +3

+3

=

0 -1

-1

=

-2 +2

0

2

3 Kayak Trips & 6 People



-2 0 +5

-4 +2 +1 -1 +6 -1

Kayak Trip 1 Kayak Trip 2 Kayak Trip 3

soft-out

-2 -1 -2 +1 +1 -1 -3



= = =

-3 +3 0 -1 -4 +2

0 -1 -2

3 Kayak Trips & 6 People

-2

-4 +2

0

+1 -1

+5

+6 -1

+ Kayak Trip 1

+ Kayak Trip 2

+ Kayak Trip 3

up

+2 +1 +1 -2 -1 +1 -2

=

-3 +3

0

=

0 -1

-1

=

-4 +2

-2

3

3 Kayak Trips & 6 People

-2

-4 +2

+

Kayak Trip 1

0

+1 -1

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

down

-1 +1 -2

-1 +1 -1

-2 -1

=

-3 +3

0

=

0 -1

-1

=

-4 +2

-2

3

3 Kayak Trips & 6 People

-3

-4 +1

+

Kayak Trip 1

+2

+1 +1

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

soft-
out

-1 +1 -2

0 +1 -1

-2 -1

=

0 +3

+3

=

-2 -1

-3

=

-3 +2

-1

3

3 Kayak Trips & 6 People



-3 +2 +5

-4 +1 +1 +1 +6 -1

Kayak Trip 1 Kayak Trip 2 Kayak Trip 3

up

+4 +2 -1 -2 -4 0 +1



0 +3 -2 -1 -3 +2

+3 -3 -1

4

3 Kayak Trips & 6 People

-3

-4 +1

+

Kayak Trip 1

+2

+1 +1

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

down

+1 0 -4

0 +1 -1

-4

=

0 +3

+3

=

-2 -1

-3

=

-3 +2

-1

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+1

+1 0

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

soft-out

+1 0 -4

+1 +3

=

+4

0 +1 -1

-3 -1

=

-4

-4 +2

-5 +2

=

-3

4

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+1

+1 0

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

up

+3

+4

0

-4

-5

-2

+1

=

+1 +3

+4

=

-3 -1

-4

=

-5 +2

-3

5

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+1

+1 0

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

down

0 +1 -3

-1 +1 -5

-1

=

+1 +3

+4

=

-3 -1

-4

=

-5 +2

-3

5

3 Kayak Trips & 6 People

-4

-4 ↓ ↑ 0

+

Kayak Trip 1

+2

+1 ↓ ↑ +2

+

Kayak Trip 2

+5

+6 ↓ ↑ -1

+

Kayak Trip 3

soft-
out

0 ↓ +1 ↓ -3 ↓

-1 ↓ +1 ↓ -1 ↓

-5 ↓

=

+1 ↓ ↑ +3

+4

=

-3 ↓ ↑ -1

-2

=

-6 ↓ ↑ +2

-4

3 Kayak Trips & 6 People

-4

-4 ↓ ↑ 0

+2

+1 ↓ ↑ +2

+5

+6 ↓ ↑ -1

+ Kayak Trip 1

+ Kayak Trip 2

+ Kayak Trip 3

up

↑ +4 ↖ +3 ↑ -1 ↑ -3 ↖ -5 ↖ -3 ↑ +1

= +1 ↓ ↑ +3

+4

= -3 ↓ ↑ -1

-2

= -6 ↓ ↑ +2

-4

3 Kayak Trips & 6 People

-4

-4 ↓ ↑ 0

+ Kayak Trip 1

+2

+1 ↓ ↑ +2

+ Kayak Trip 2

+5

+6 ↓ ↑ -1

+ Kayak Trip 3

down

+1 ↓ +1 ↓ -4 ↓

-1 ↓ +1 ↓ -1 ↓

-5 ↓

=

+1 ↓ ↑ +3

+4

=

-3 ↓ ↑ -1

-2

=

-6 ↓ ↑ +2

-4

6

3 Kayak Trips & 6 People



-5 +4 +5

-4 -1 +1 +3 +6 -1

Kayak Trip 1 | Kayak Trip 2 | Kayak Trip 3

soft-out

+1 +1 -4 -1 +1 -1 -5



+2 +3 -4 -1 -6 +2

+5 -5 -4

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+4

+1 +3

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

up

+4 +4

-1 -4

-3 -6

+1

=

+2 +3

+5

=

-4 -1

-5

=

-6 +2

-4

7

3 Kayak Trips & 6 People

-5 +4 +5

-4 -1 +1 +3 +6 -1

Kayak Trip 1 Kayak Trip 2 Kayak Trip 3

down +1 +1 -4 -1 +1 -1 -6

+2 +3 -4 -1 -6 +2

+5 -5 -4

7

3 Kayak Trips & 6 People

-5 +4 +5

-4 -1 +1 +3 +6 -1

Kayak Trip 1 | Kayak Trip 2 | Kayak Trip 3

soft-out

+1 +1 -4 -1 +1 -1 -6

+2 +3 +5

-4 -1 -5

-7 +2 -5

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+4

+1 +3

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

up

+4 +4

-1 -4

-4 -6

+1

=

+2 +3

+5

=

-4 -1

-5

=

-7 +2

-5

8

3 Kayak Trips & 6 People

-5

-4 ↓ ↑ -1

+ Kayak Trip 1

+4

+1 ↓ ↑ +3

+ Kayak Trip 2

+5

+6 ↓ ↑ -1

+ Kayak Trip 3

down

+1 ↓ +1 ↓ -4 ↓

-1 ↓ +1 ↓ -1 ↓

-6 ↓

=

+2 ↓ ↑ +3

+5

=

-4 ↓ ↑ -1

-5

=

-7 ↓ ↑ +2

-5

8

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+5

+1 +4

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

soft-out

+1 +1 -4

-1 +1 -1

-6

=

+2 +3

+5

=

-4 -1

-5

=

-7 +2

-5

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+5

+1 +4

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

up

+4 +4 -1 -4 -4 +1 -6

=

+2 +3

+5

=

-4 -1

-5

=

-7 +2

-5

9

3 Kayak Trips & 6 People

-5 +5 +5

-4 -1 +1 +4 +6 -1

Kayak Trip 1 Kayak Trip 2 Kayak Trip 3

down +1 +1 -4 -1 +1 -1 -6

= = =

+2 +3 -4 -1 -7 +2

+5 -5 -5

3 Kayak Trips & 6 People

-5 +5 +5

-4 -1 +1 +4 +6 -1

Kayak Trip 1 | Kayak Trip 2 | Kayak Trip 3

soft-out

+1 +1 -4 -1 +1 -1 -6

= = =

+2 +3 -4 -1 -7 +2

+5 -5 -5

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+5

+1 +4

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

up

+4 +4 -1 -4 -4 +1 -6

=

+2 +3

+5

=

-4 -1

-5

=

-7 +2

-5

10

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+5

+1 +4

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

down

+1 +1 -4

-1 +1 -1

-6

=

+2 +3

+5

=

-4 -1

-5

=

-7 +2

-5

10

3 Kayak Trips & 6 People

-5

-4 -1

+

Kayak Trip 1

+5

+1 +4

+

Kayak Trip 2

+5

+6 -1

+

Kayak Trip 3

soft-
out

+1

+1

-4

-1

+1

-1

-6

=

+2 +3

+5

=

-4 -1

-5

=

-7 +2

-5

3 Kayak Trips & 6 People

convergence!!

-5

-4 -1

+

+5

+1 +4

+

+5

+6 -1

+

+1 +4

+1 +4

-4 -1

-1 -4

+1 -6

-1 -4

-6 +1

=

+2 +3

+5

=

-4 -1

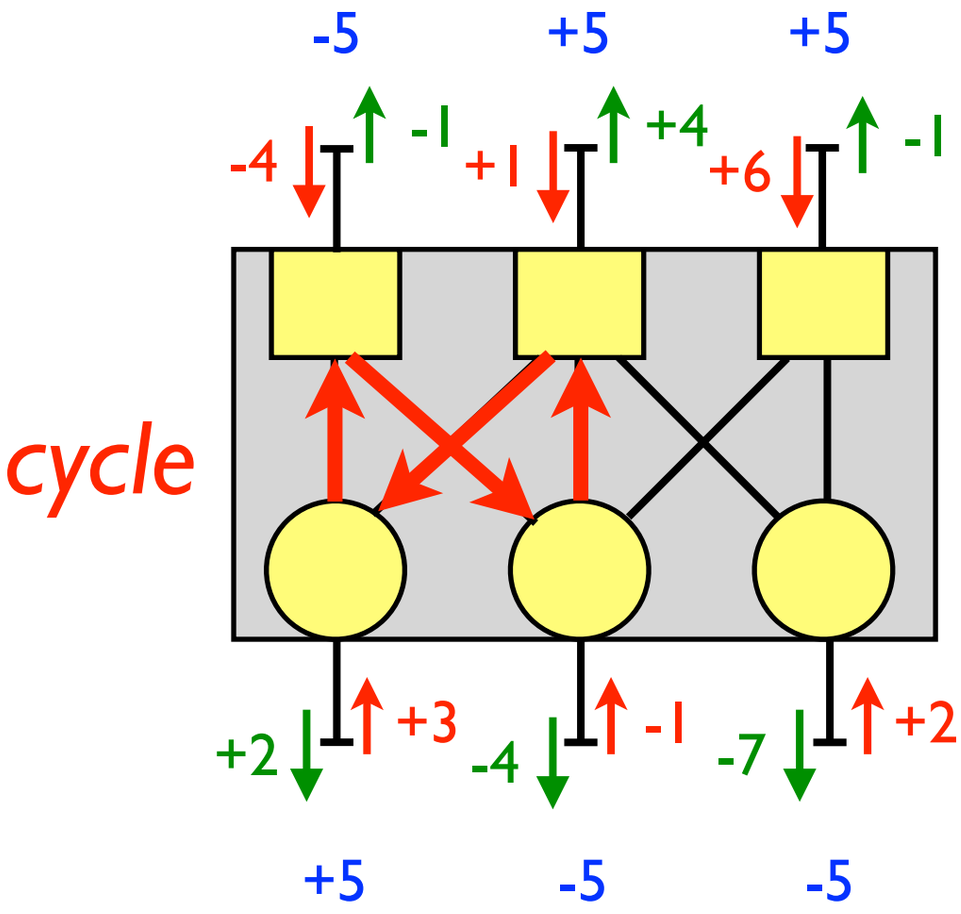
-5

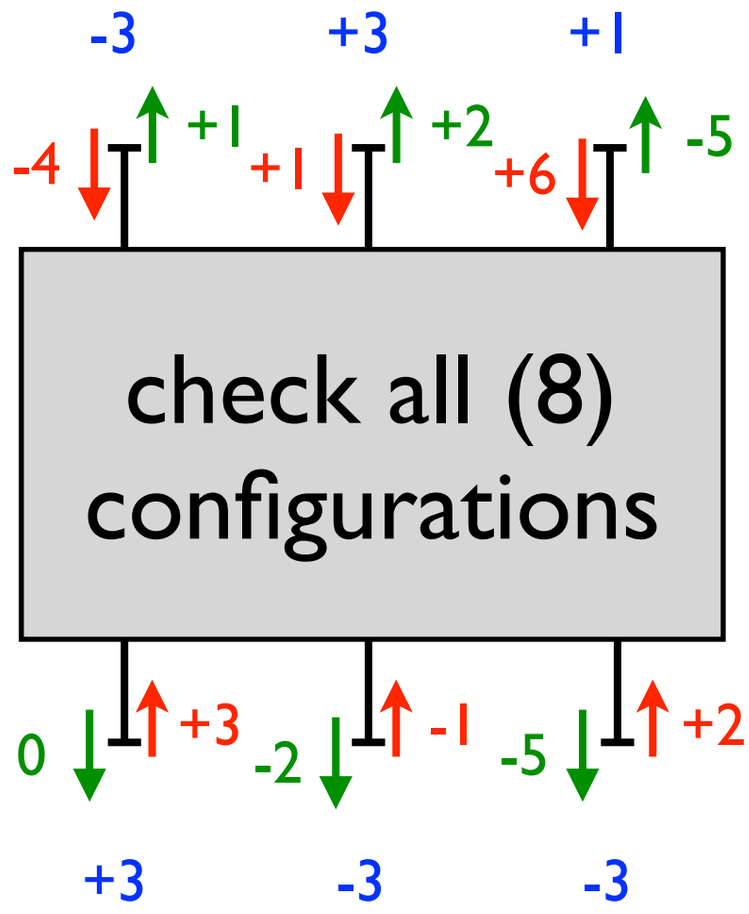
=

-7 +2

-5

Is it Optimal/Same as Direct SISO?



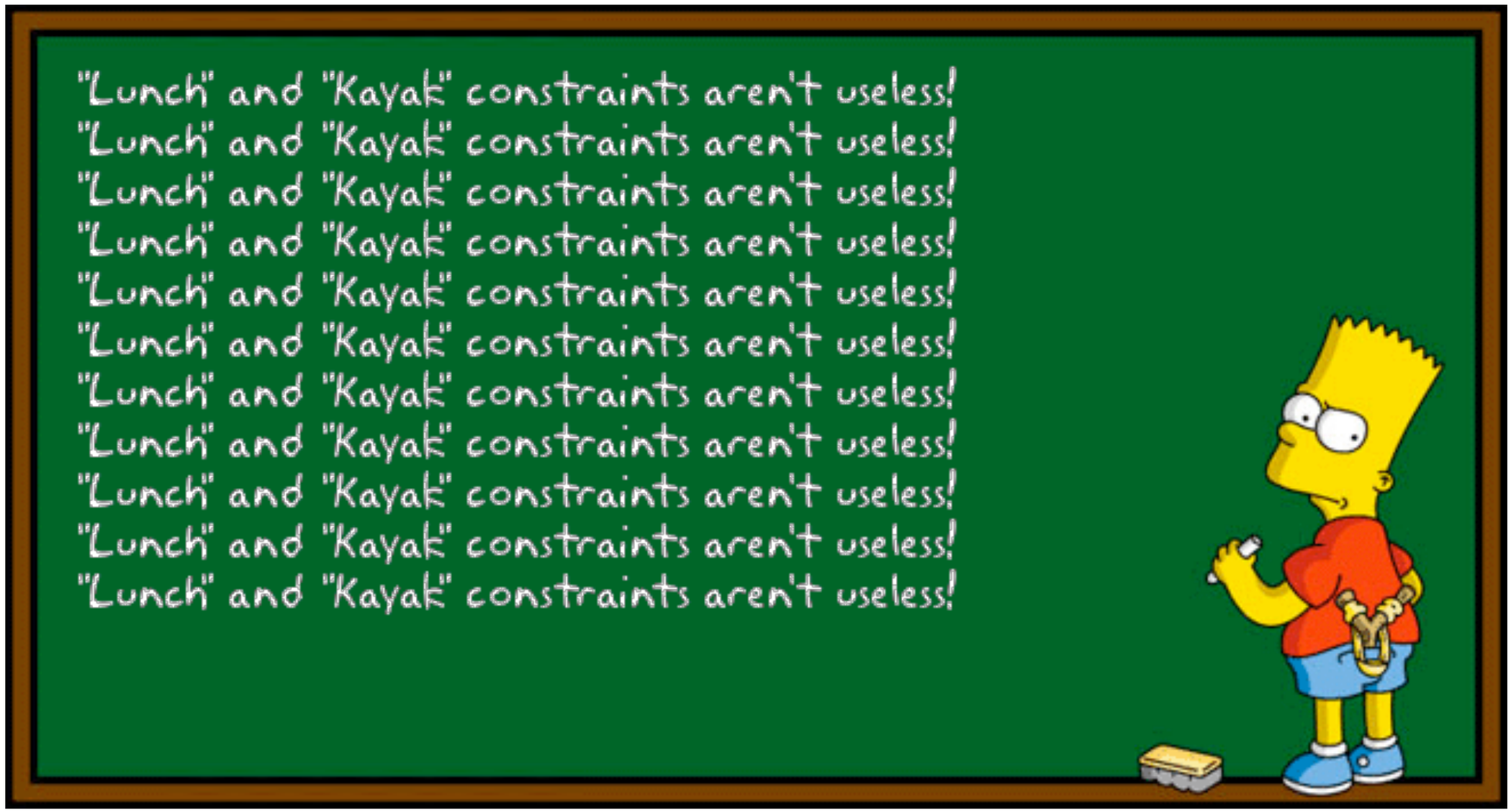


Why did this not "work"?

Message-passing on graphs is optimal if there are no cycles in the graphical model!!!

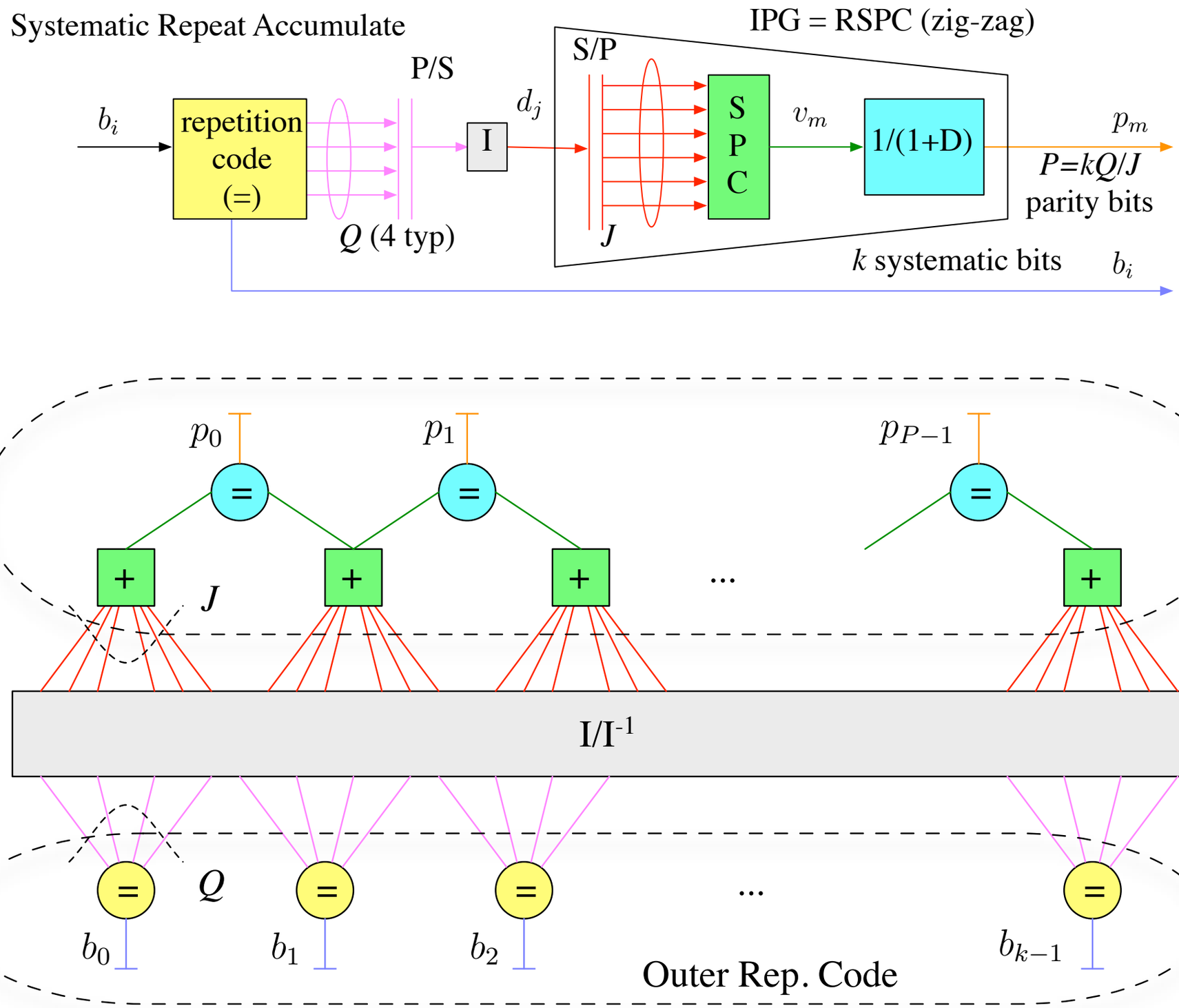
"Iterative Message Passing"

- Message passing on the graph with cycles is not optimal, but it can be a very effective, low complexity approximation
  - This is iterative decoding/detection
    - Turbo decoding, LDPC decoding, etc.
  - In contrast to the example, messages may never converge!
    - Stop when messages don't change much!

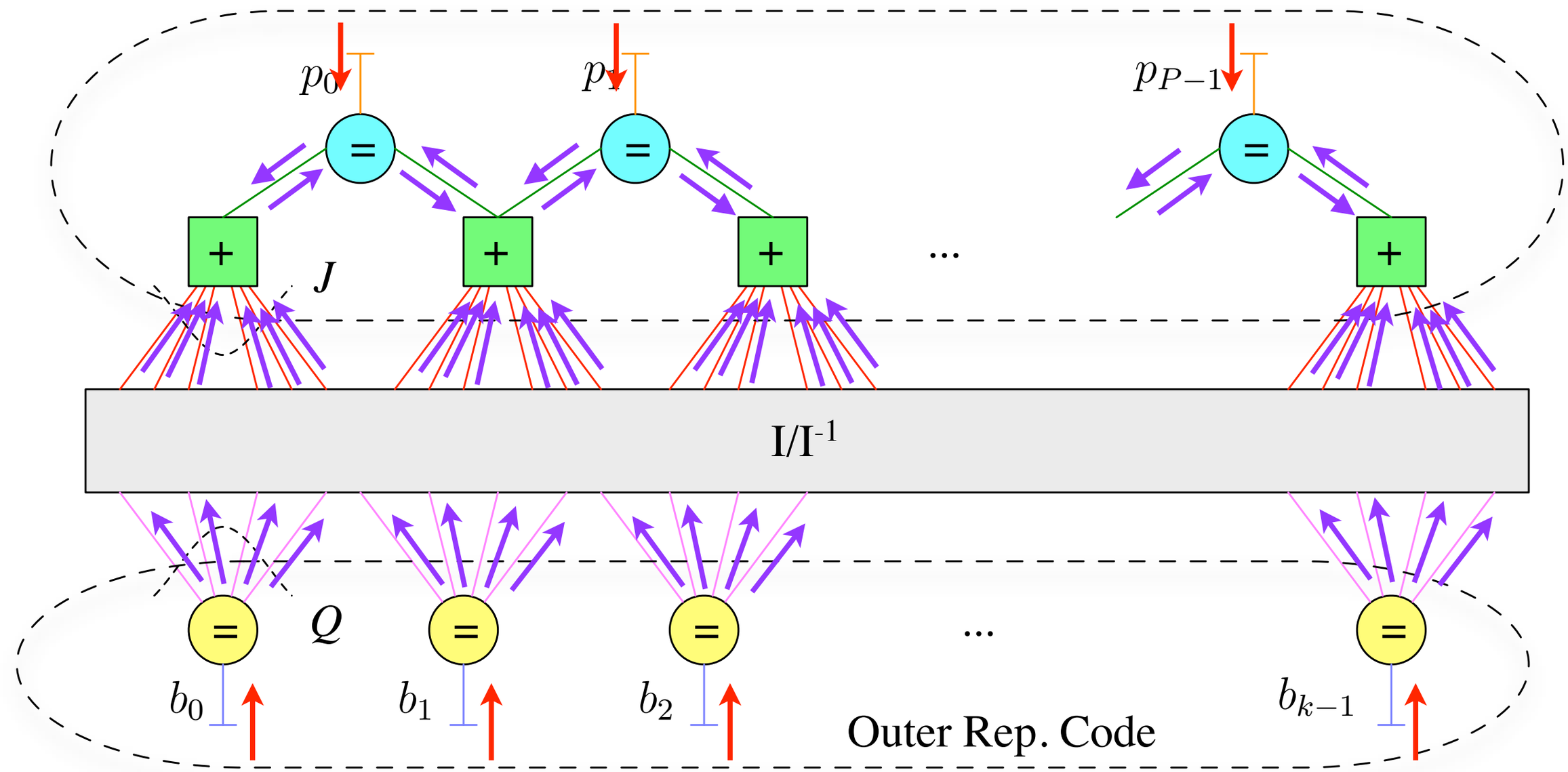


Iterative Decoding of Systematic RA Code

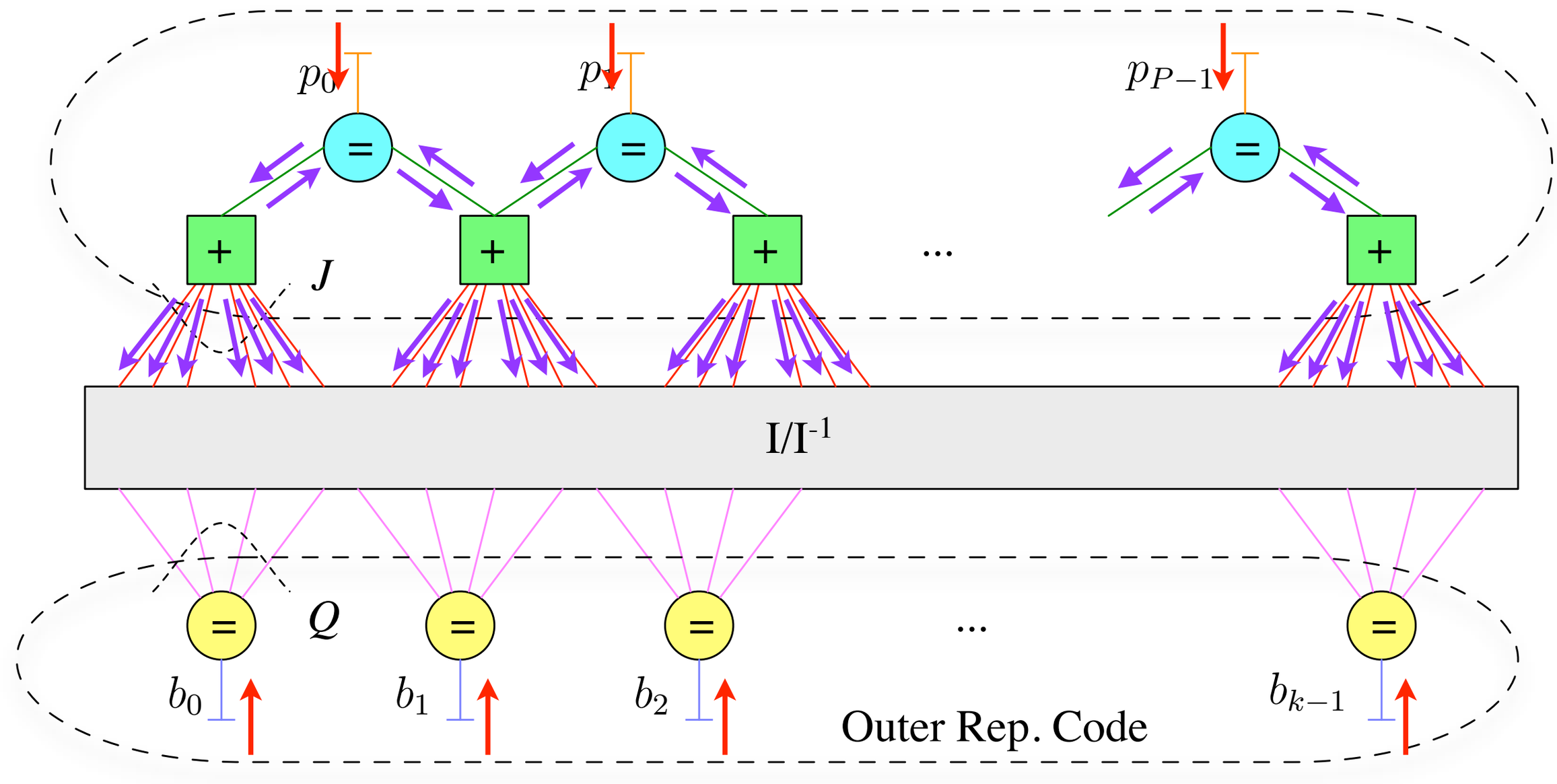
Systematic RA Code



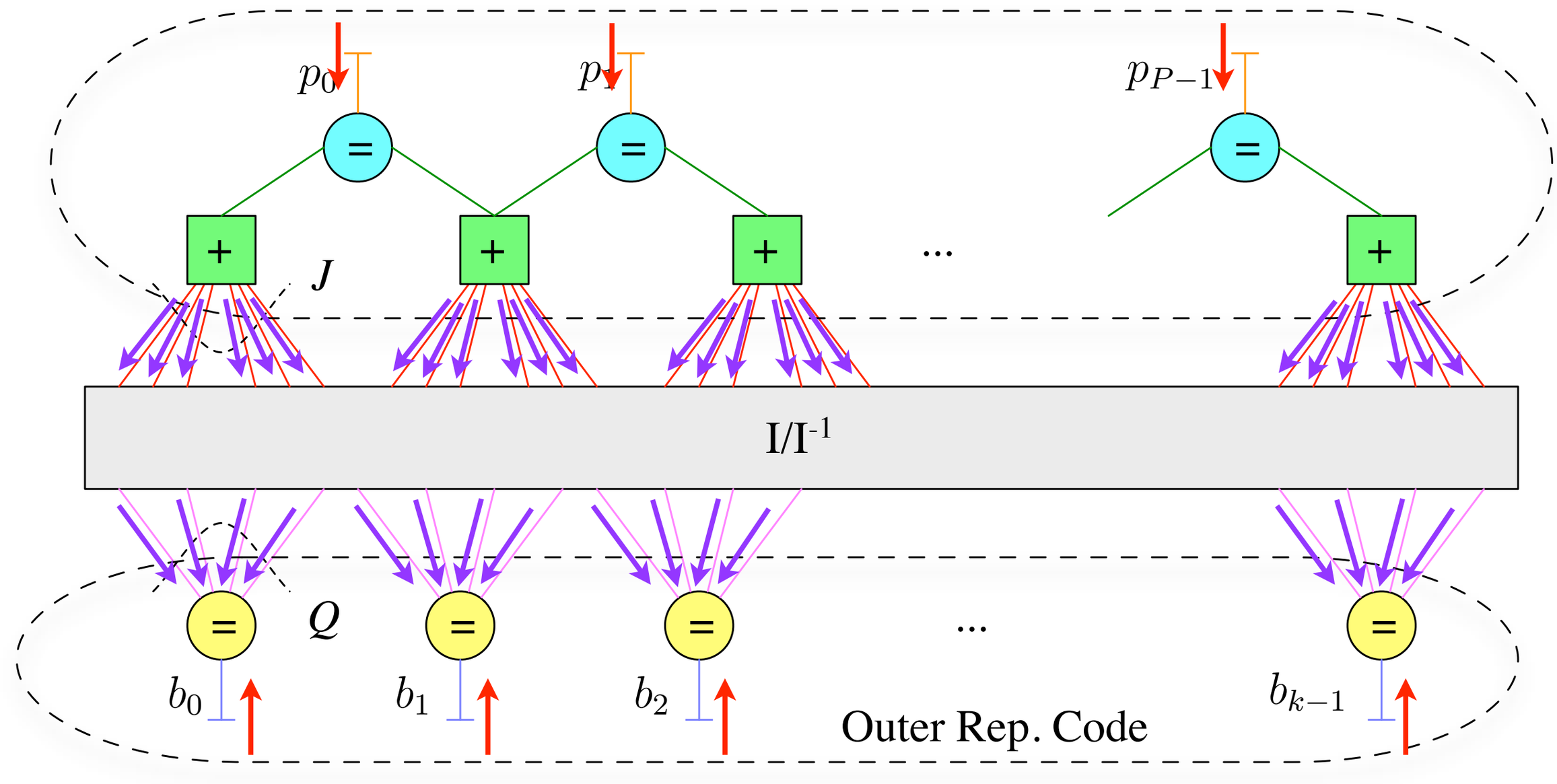
Decoding of Systematic RA Code



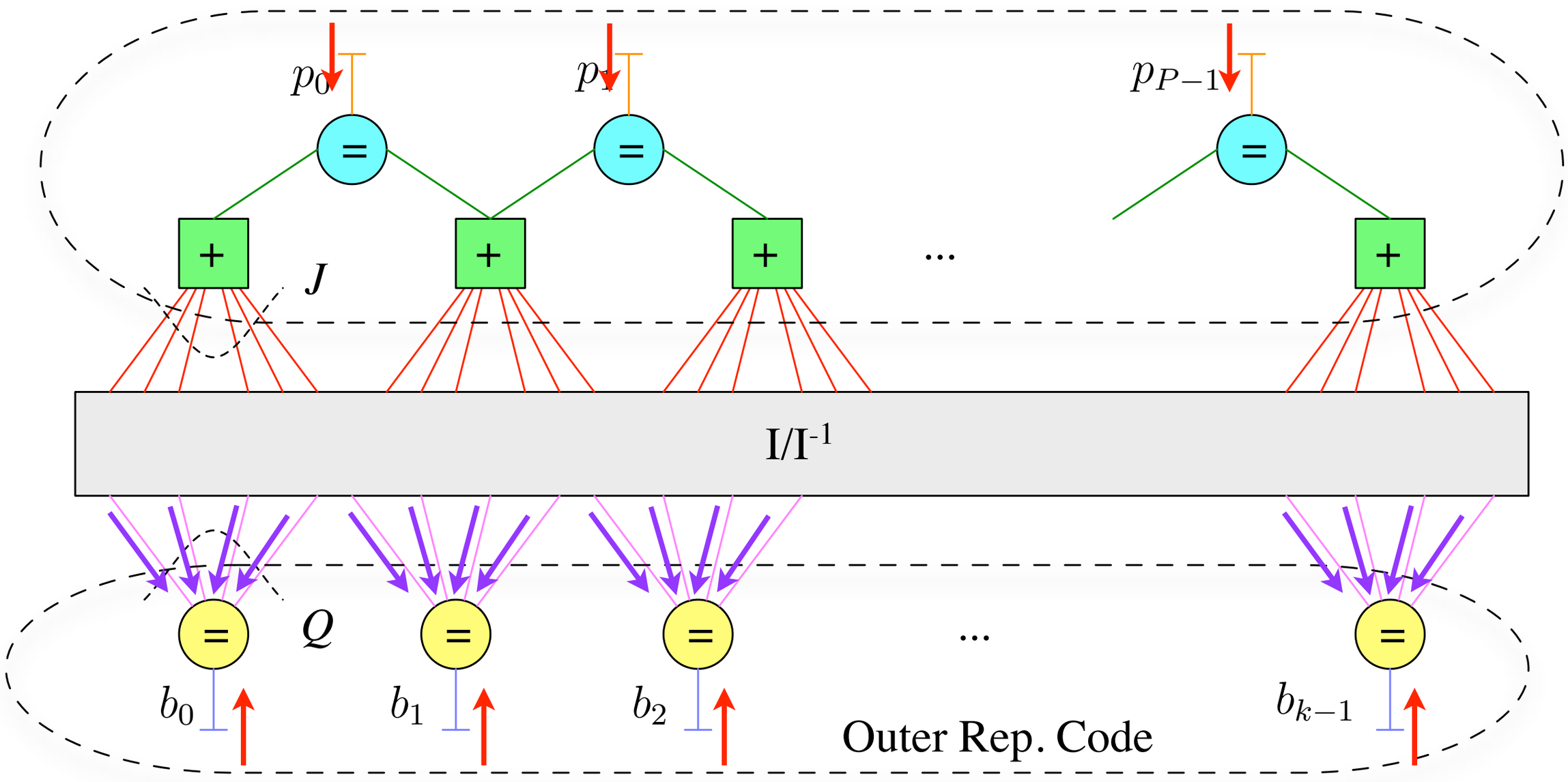
Decoding of Systematic RA Code



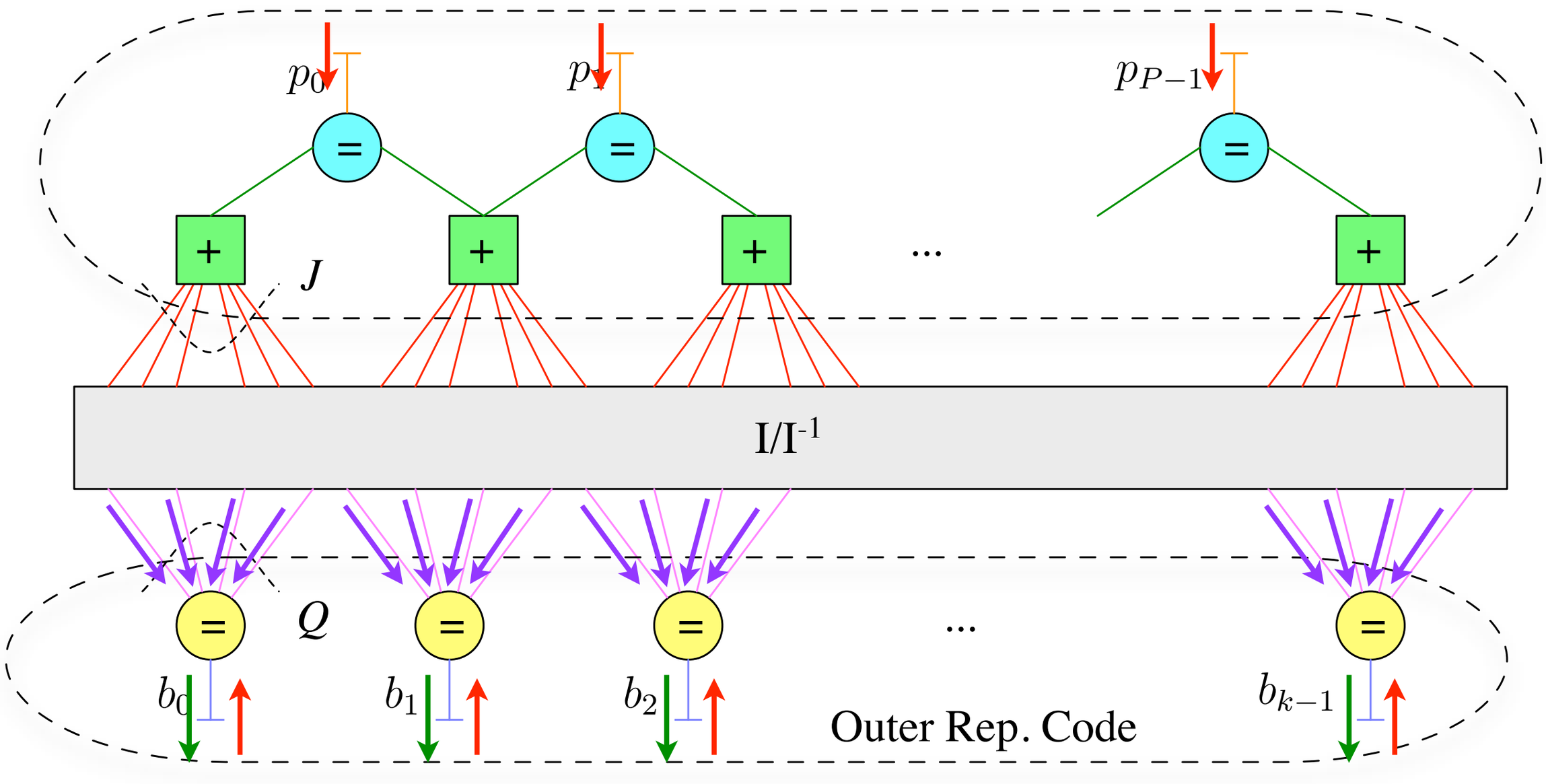
Decoding of Systematic RA Code



Decoding of Systematic RA Code

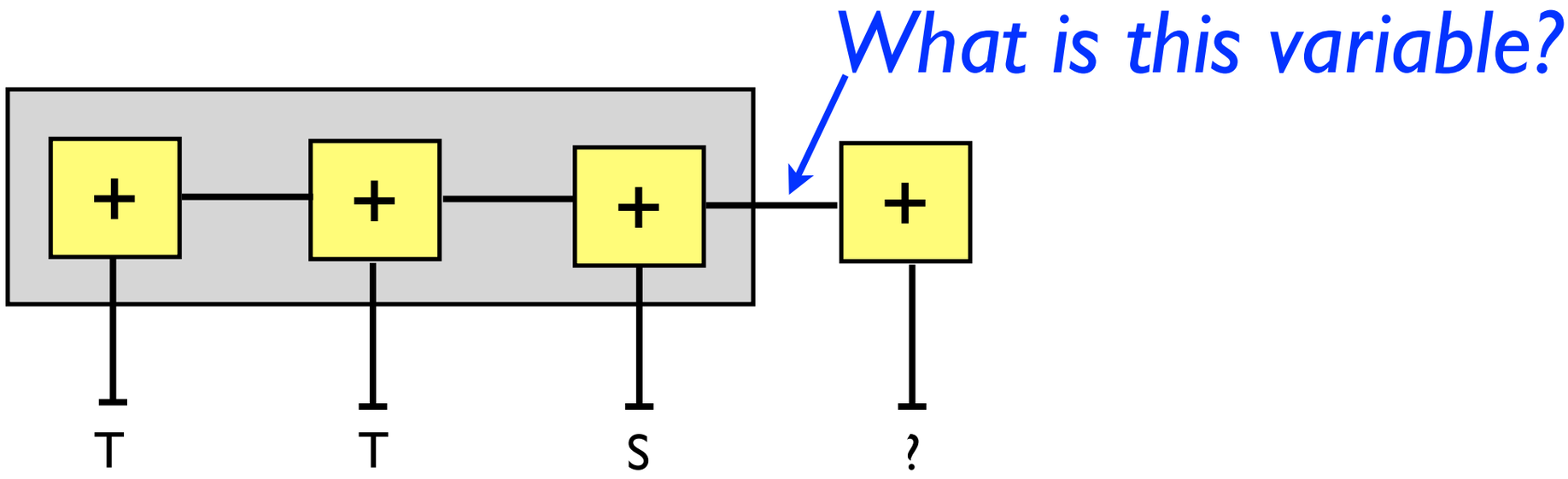


Decoding of Systematic RA Code



Trellis View of Processing

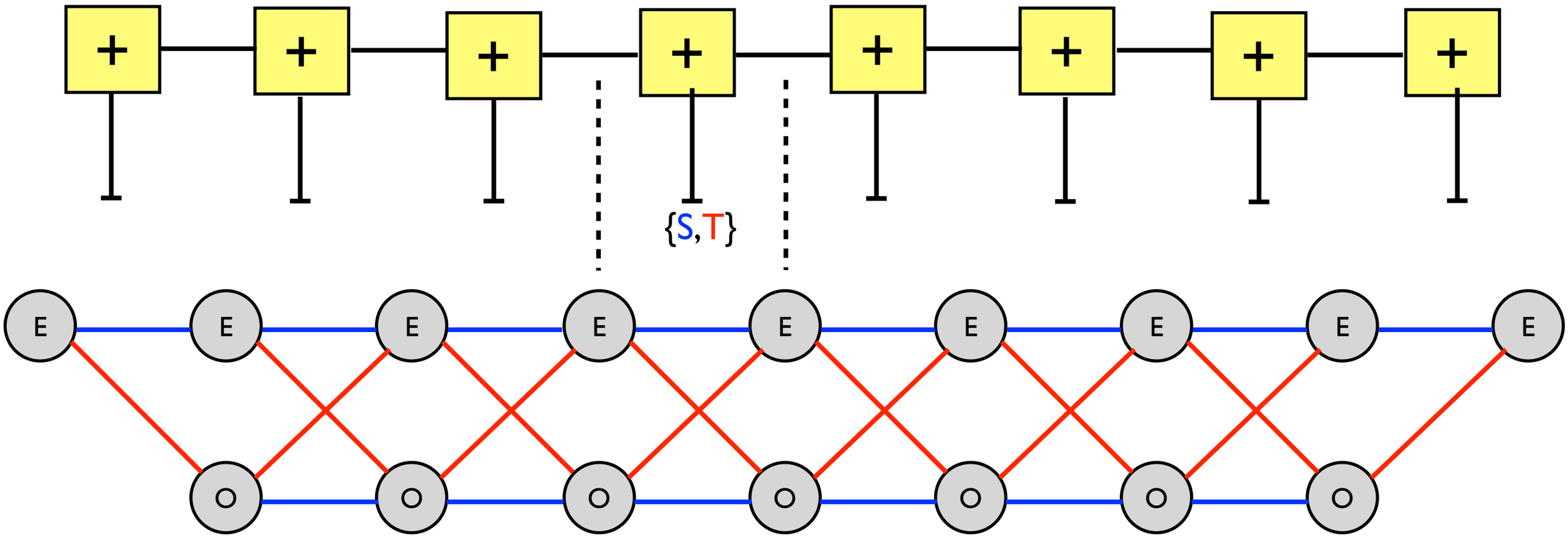
Trellis Diagrams



It is the "state" of the group to your left

- *Even or Odd number of T's -- {E,O}*

Trellis Diagrams

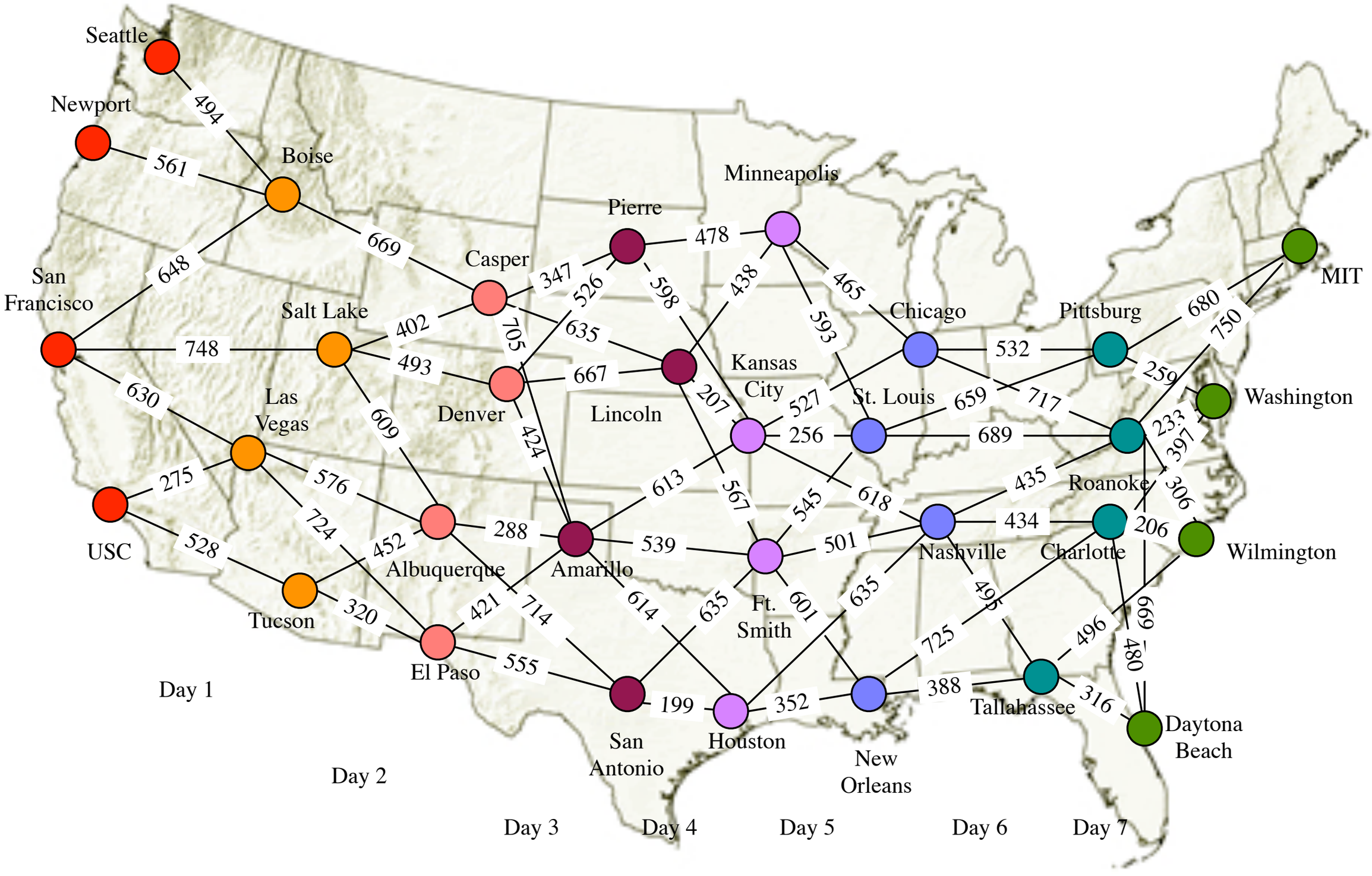


The SISO computation is related to shortest paths in this trellis

-e.g., the Forward-Backward Algorithm

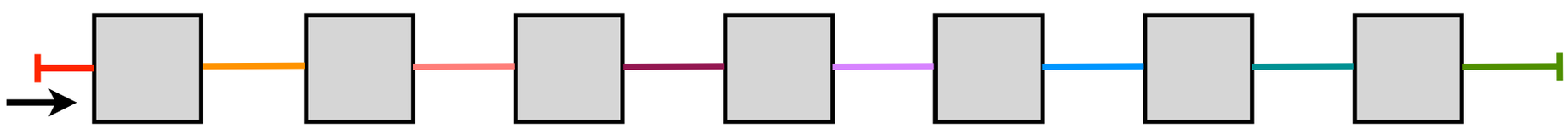
Viterbi & Forward Backward Algorithm Details

Road Trip Example

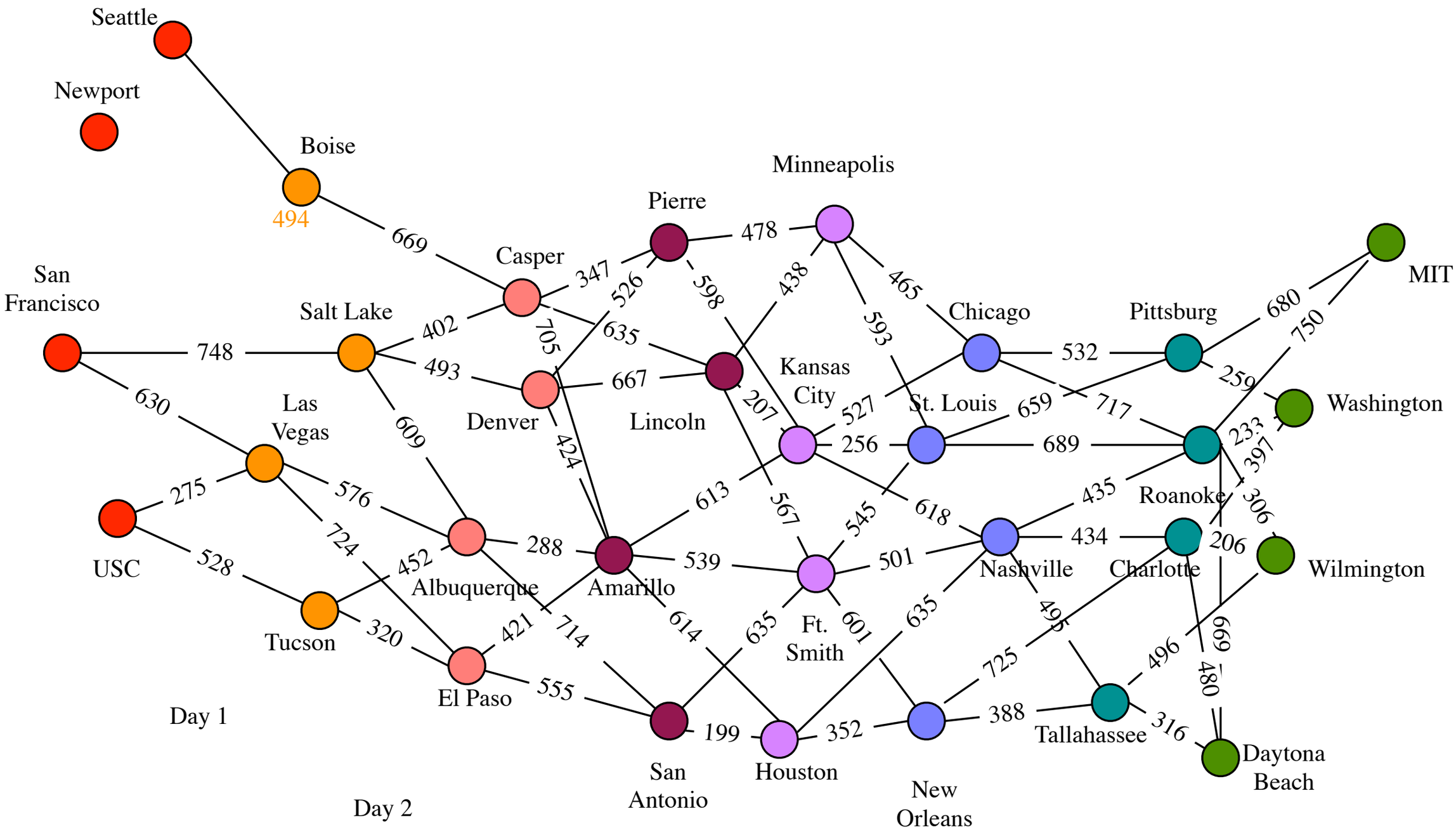


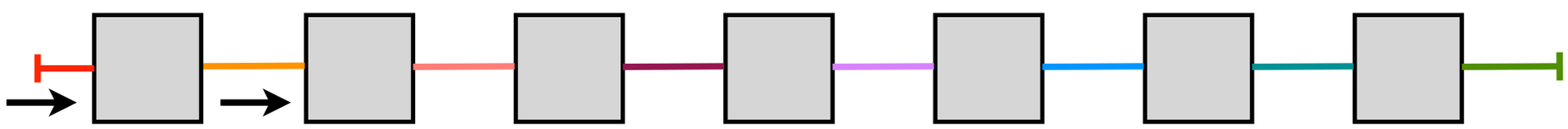
Seattle
Newport
494
561
Boise
669
648
San Francisco
748
630
Salt Lake
402
493
609
Las Vegas
275
576
724
USC
528
452
Albuquerque
288
Tucson
320
421
714
El Paso
555
Casper
347
526
705
635
Denver
424
Pierre
478
598
667
Lincoln
207
613
567
Minneapolis
438
465
593
Kansas City
527
256
545
618
501
Ft. Smith
601
635
Amarillo
539
614
635
199
San Antonio
352
Houston
Chicago
532
717
659
St. Louis
689
435
434
Nashville
725
388
New Orleans
Pittsburg
680
259
750
233
397
306
206
Roanoke
Charlotte
Wilmington
MIT
Washington
496
669
480
316
Tallahassee
Daytona Beach

Day 1
Day 2
Day 3
Day 4
Day 5
Day 6
Day 7

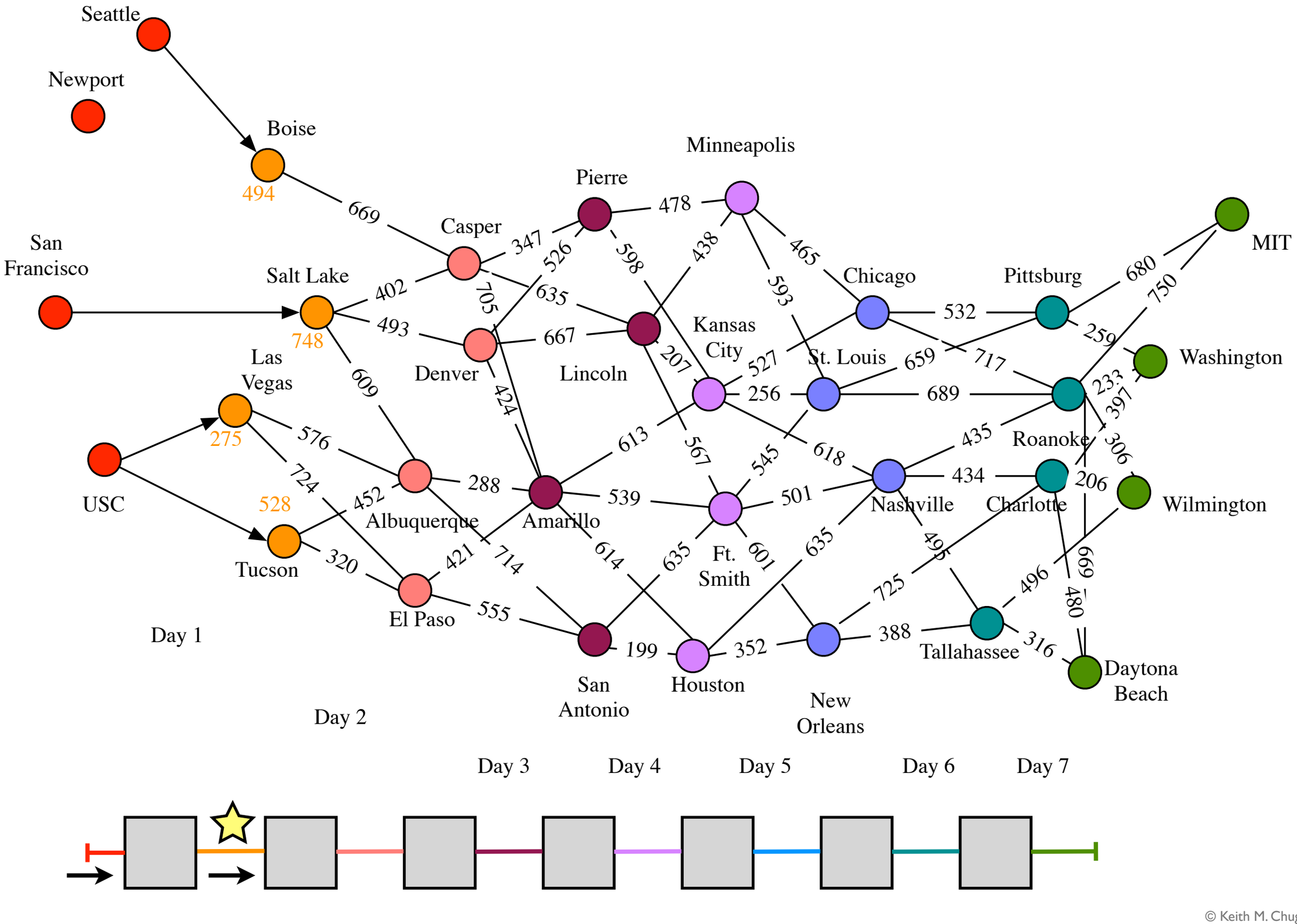


If best path from west coast to east coast, passes through Boise, it must coincide with best path from west coast to Boise

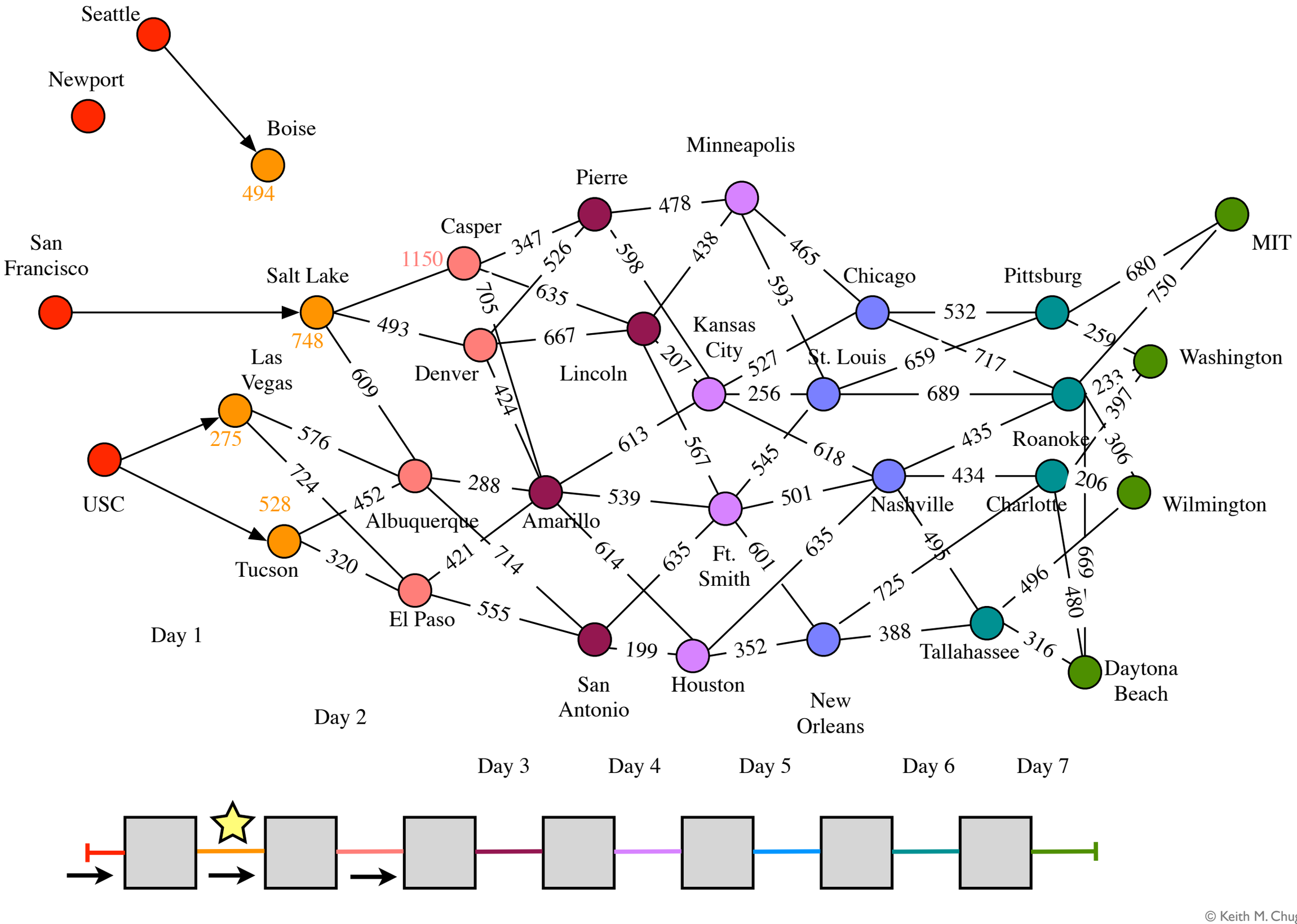




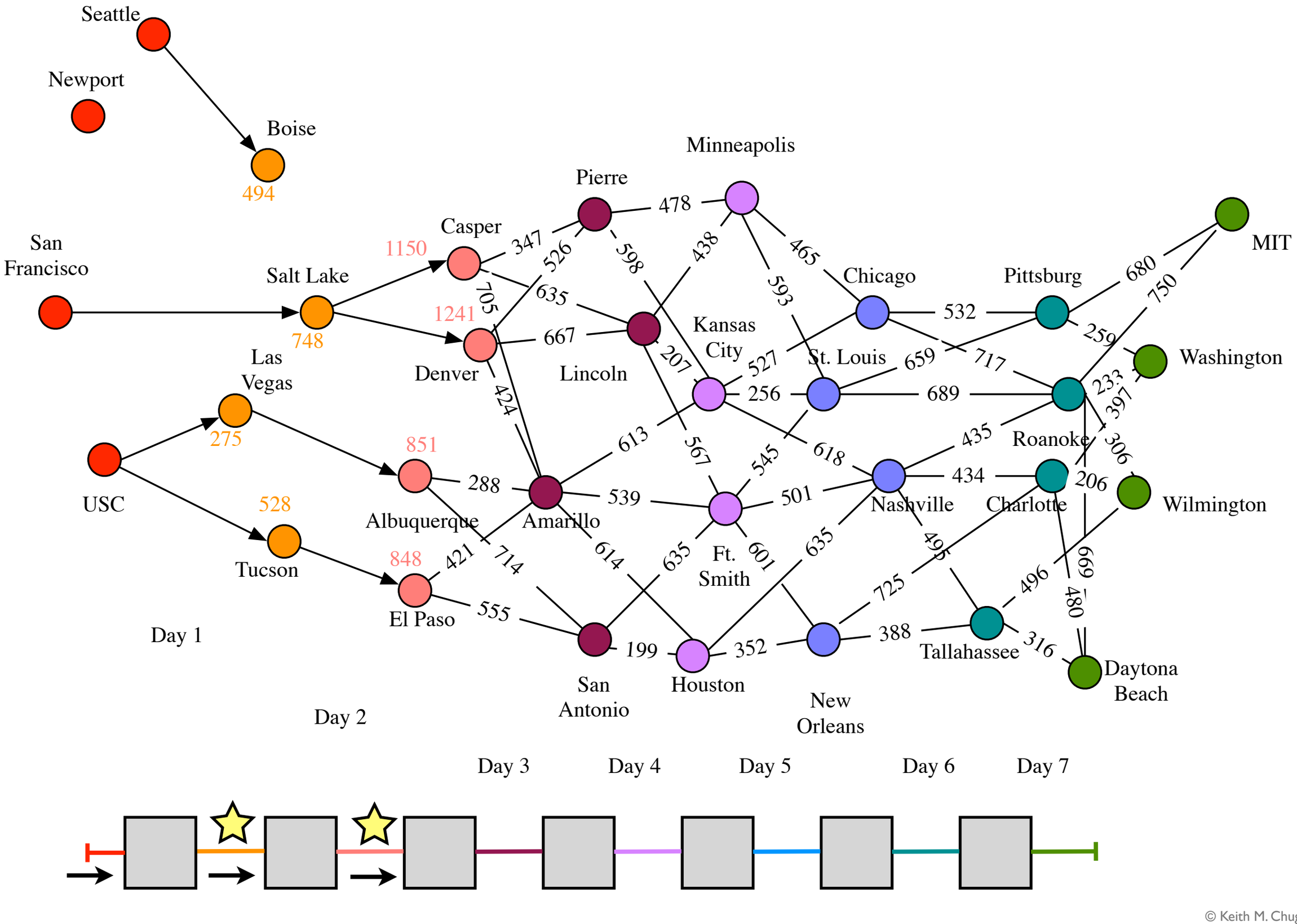
Repeat Boise-argument for Salt Lake, Las Vegas, Tucson



Repeat for Casper



Repeat for day-2 destination cities



Day 1

Day 2

Day 3

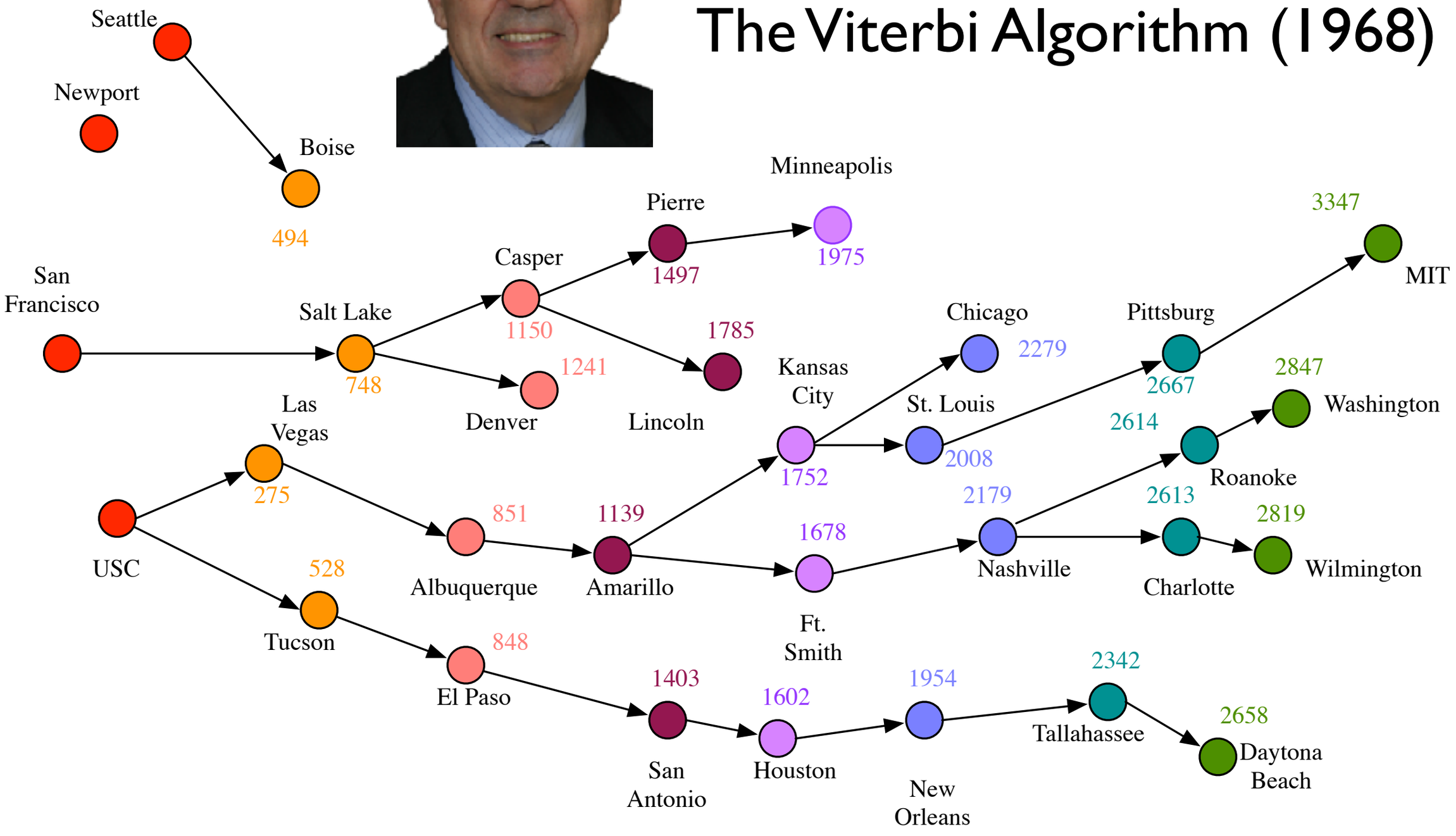
Day 4

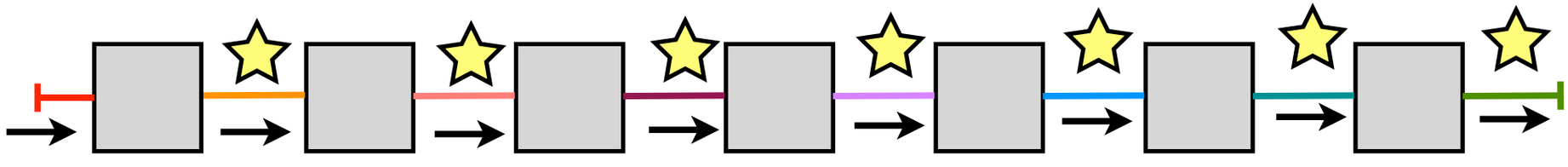
Day 5

Day 6

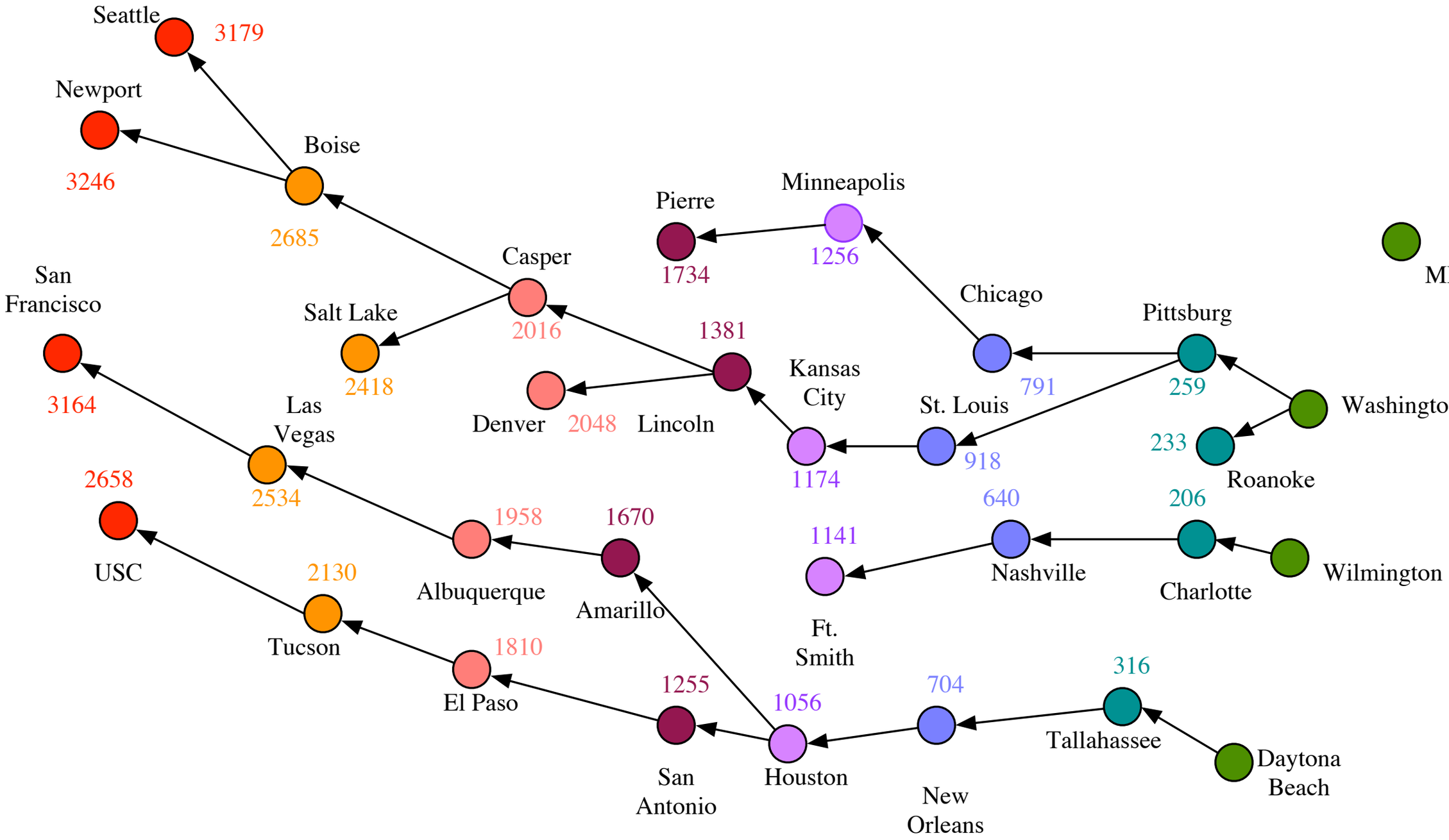
Day 7

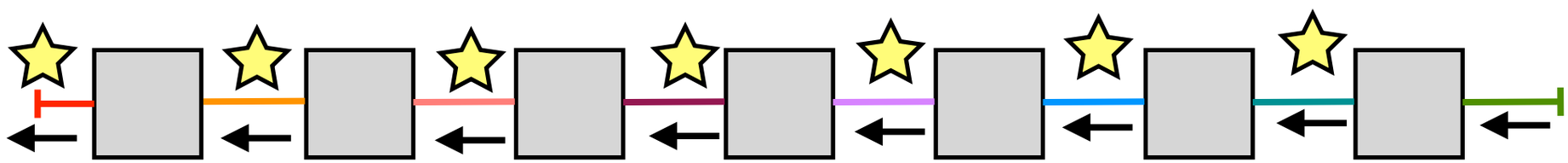
Andrew Viterbi
The Viterbi Algorithm (1968)



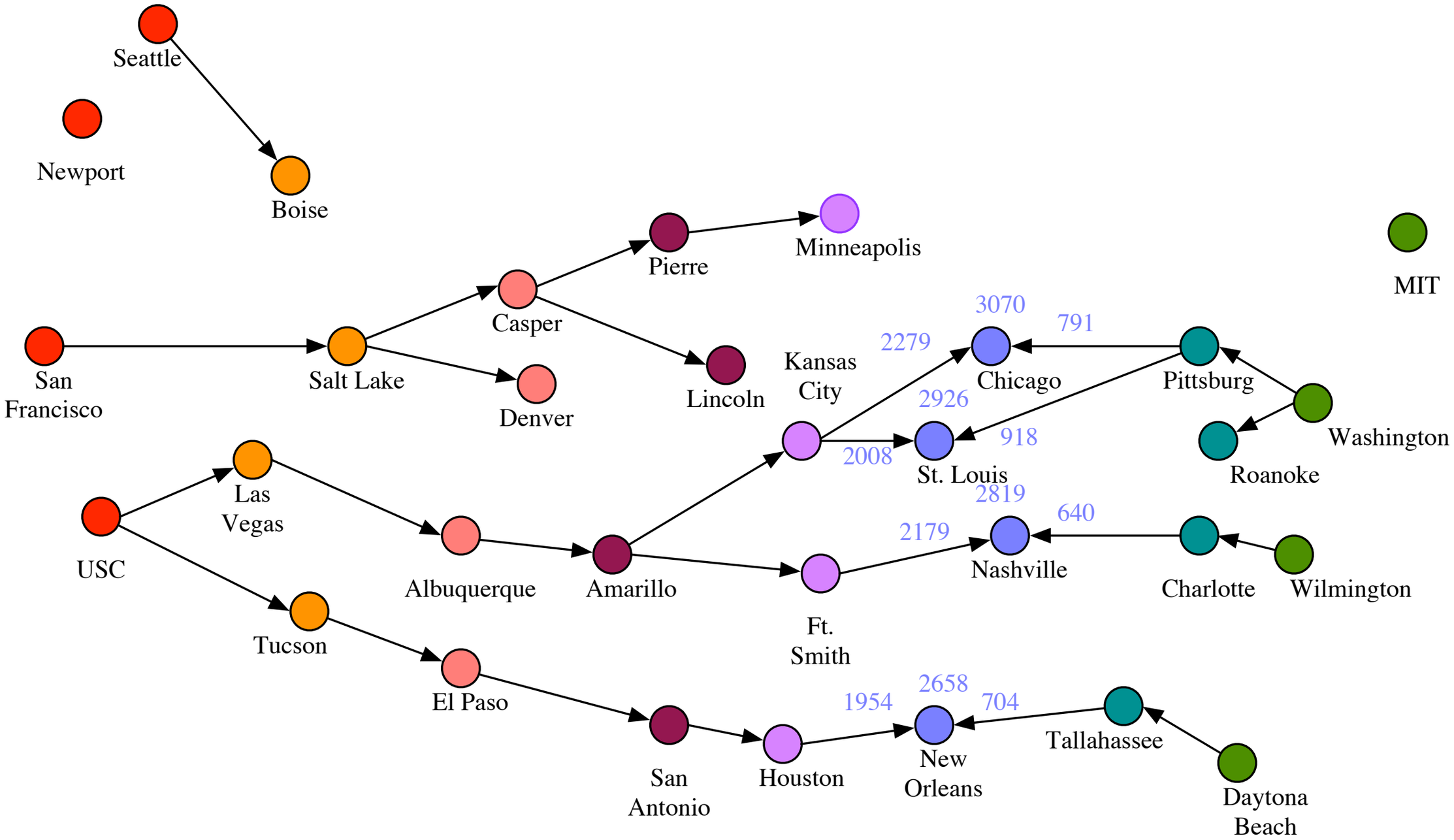


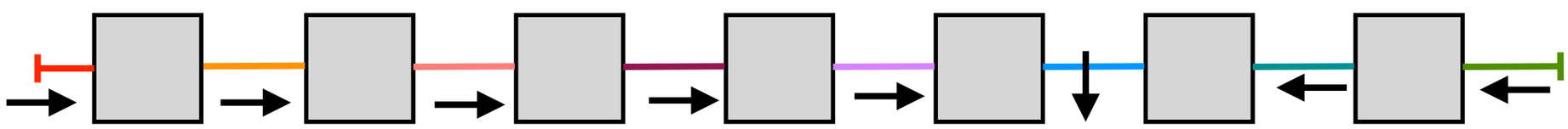
This algorithm could have been developed based on East-to-West argument



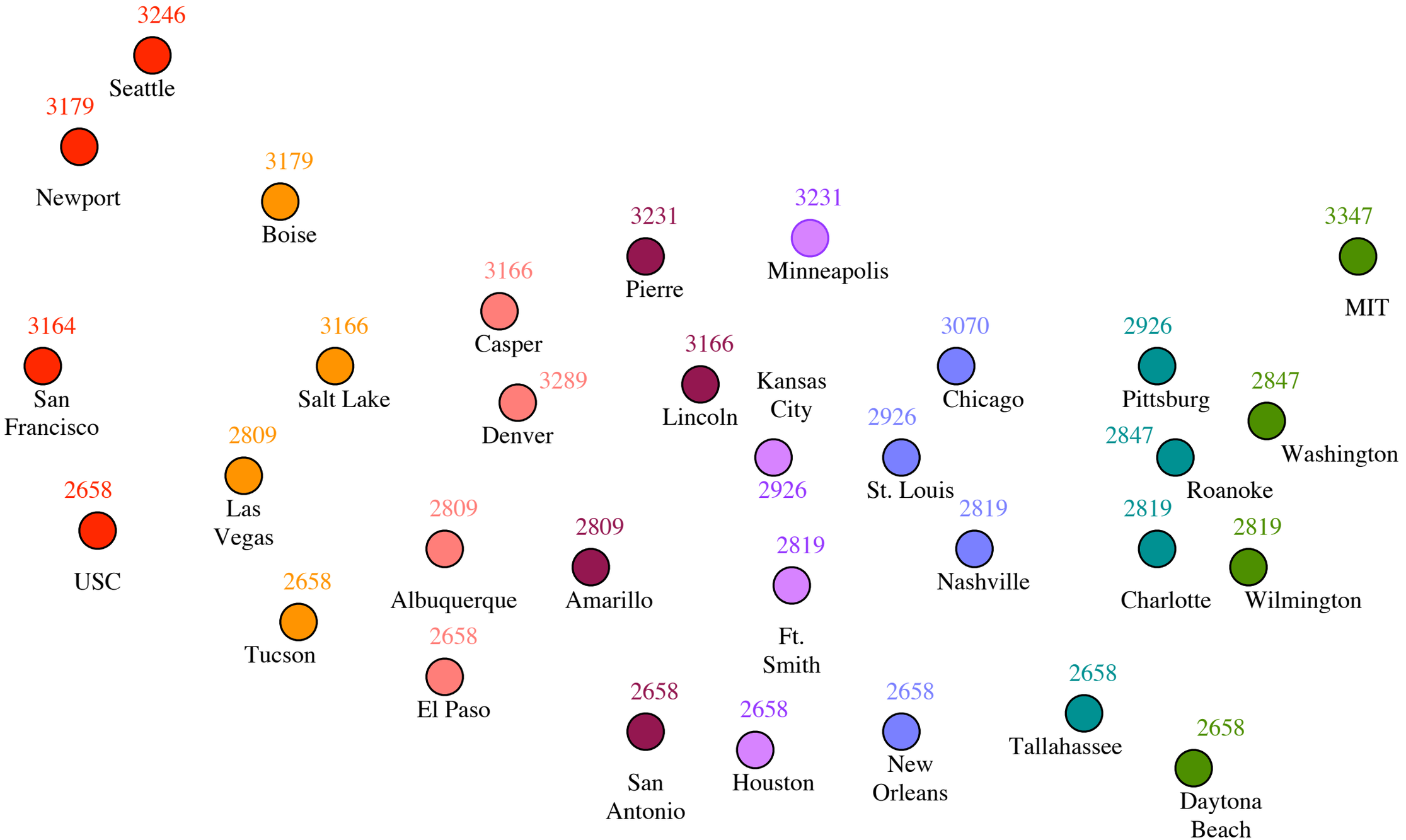


What is the shortest path going through each of the "blue" cities?



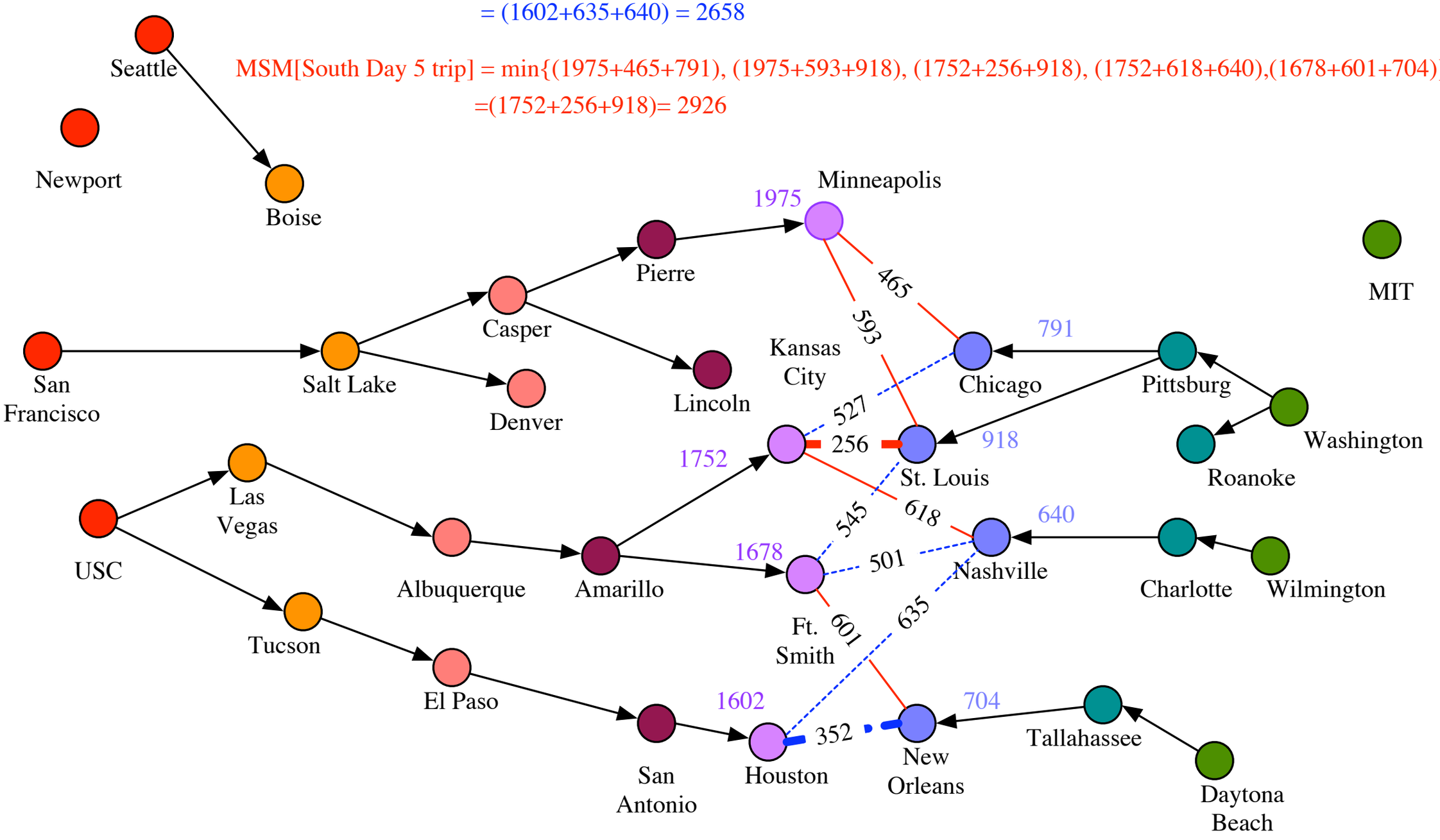


MSM[city] = length of shortest path that passes through that city



MSM[North Day 5 trip] = min{(1752+527+791), (1678+545+918), (1678+501+640), (1602+635+640),(1602+352+704)}
= (1602+635+640) = 2658

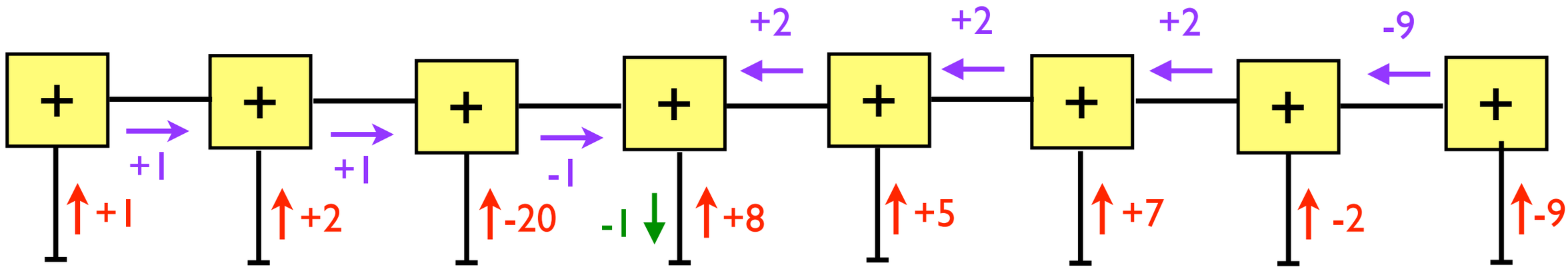
MSM[South Day 5 trip] = min{(1975+465+791), (1975+593+918), (1752+256+918), (1752+618+640),(1678+601+704)}
=(1752+256+918)= 2926

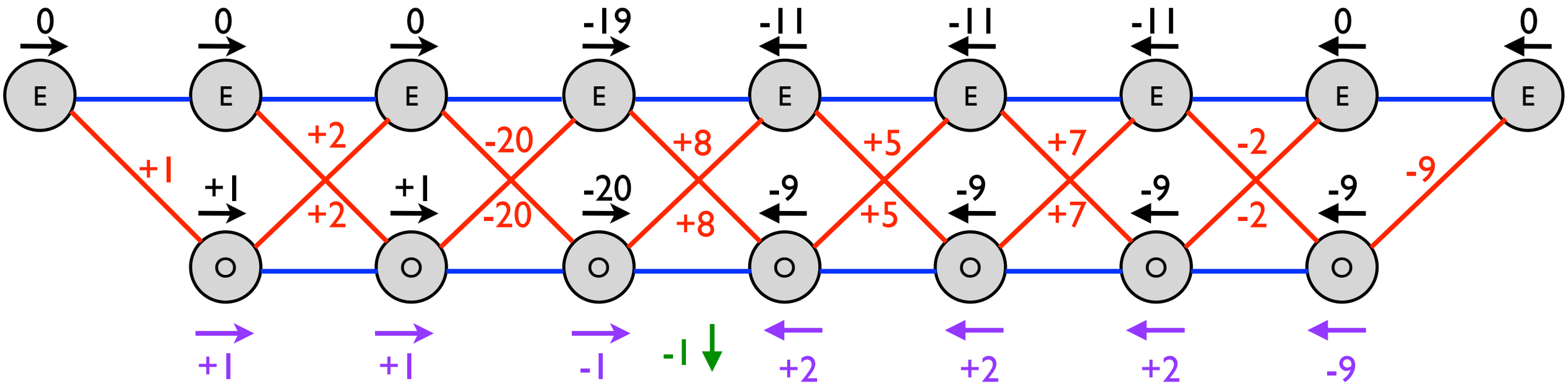


shortest path thru red edge = 2926

shortest path thru blue edge = 2658

Message Passing / FBA

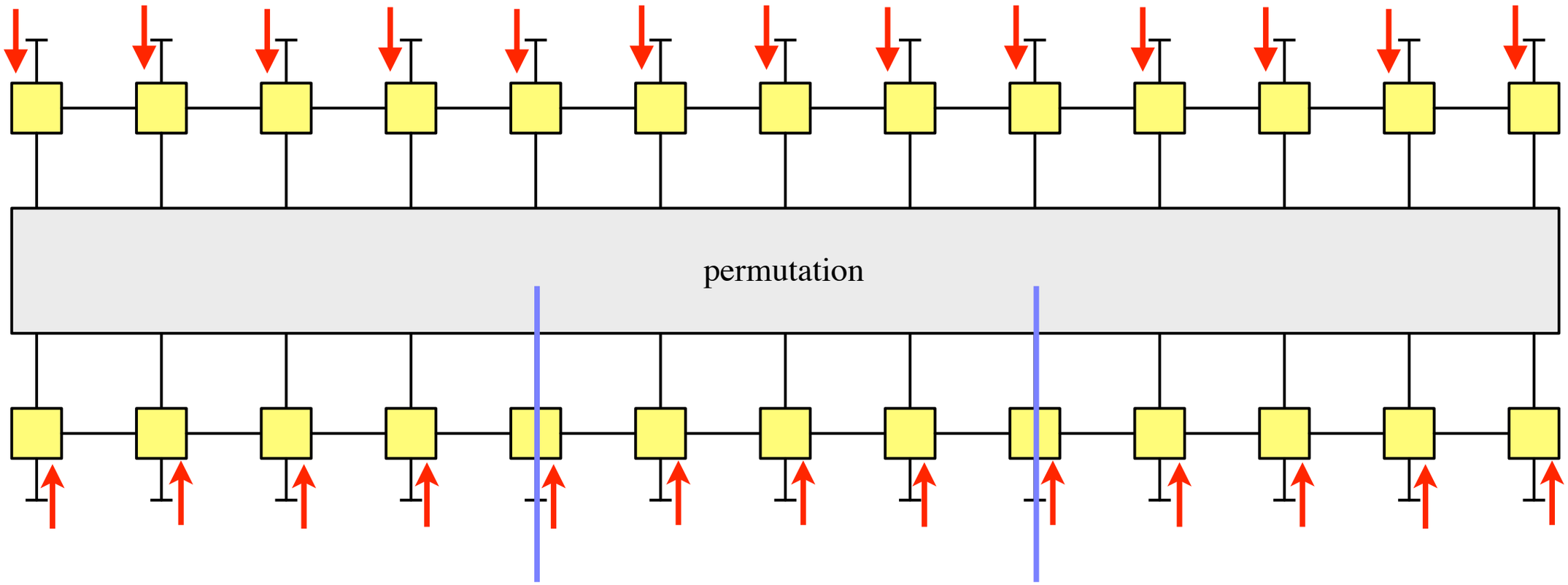




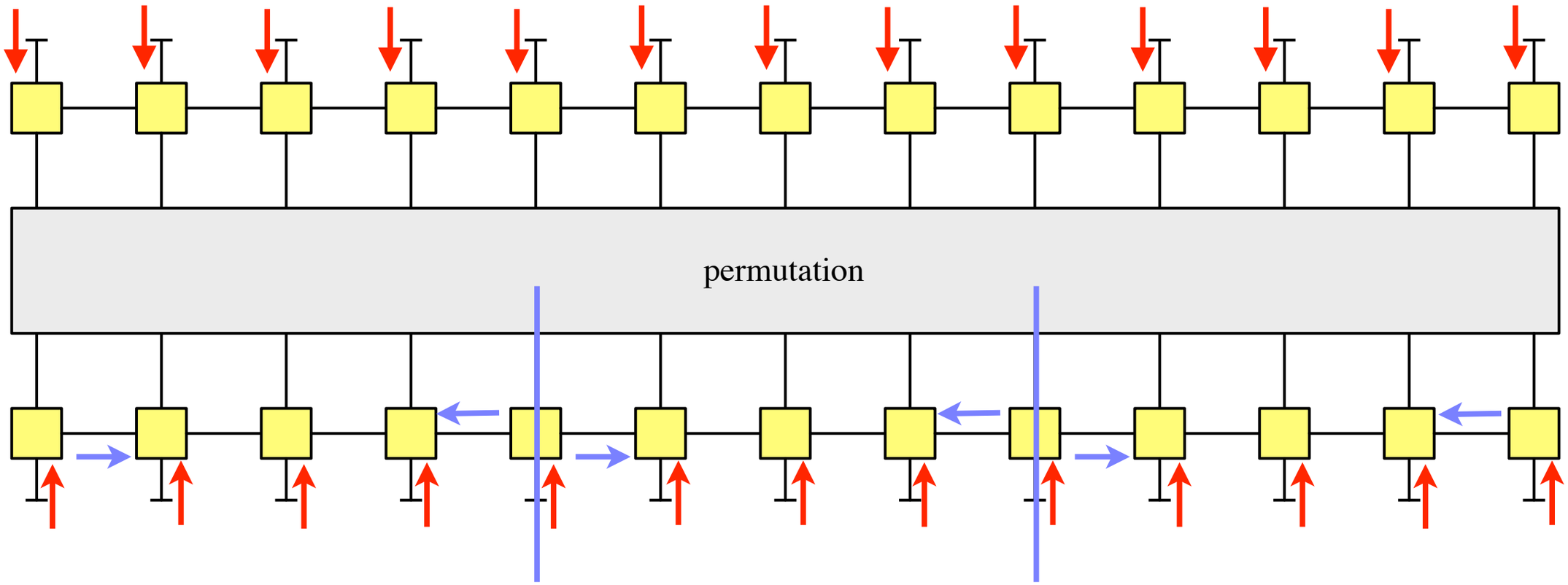
Example Schedule for Iterative Decoding

Parallel Decoding Architecture

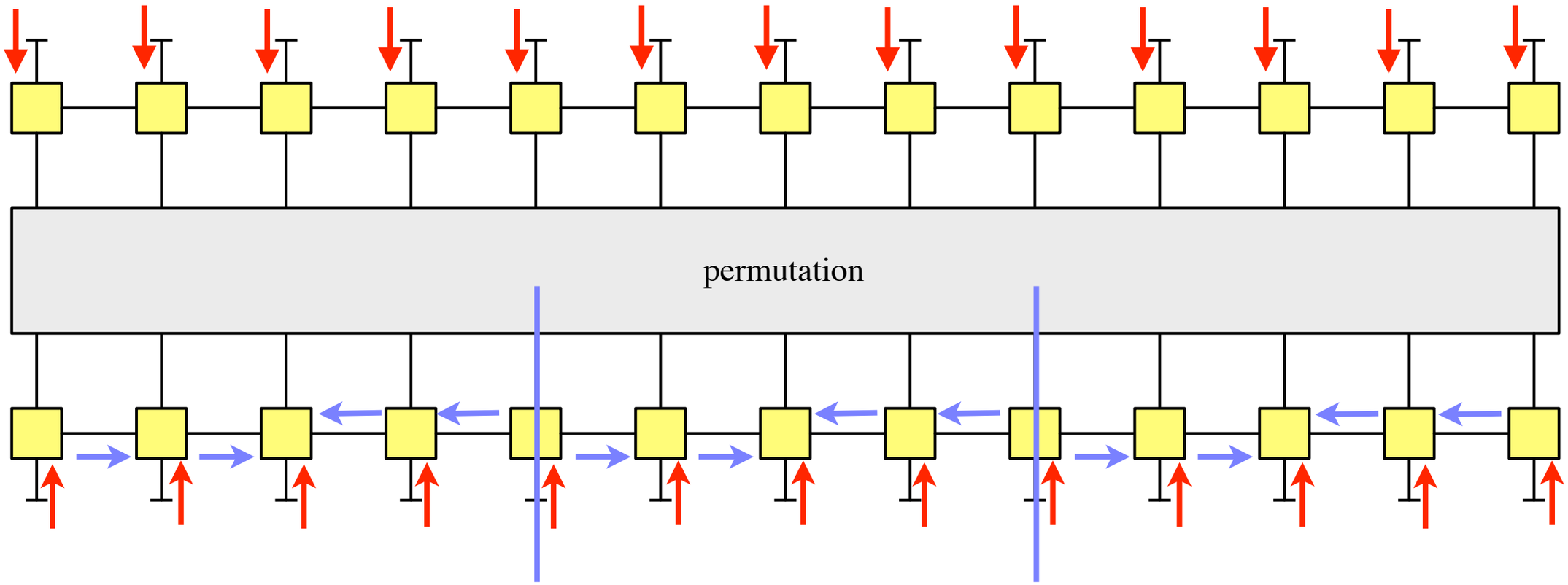
Typical Iterative Schedule



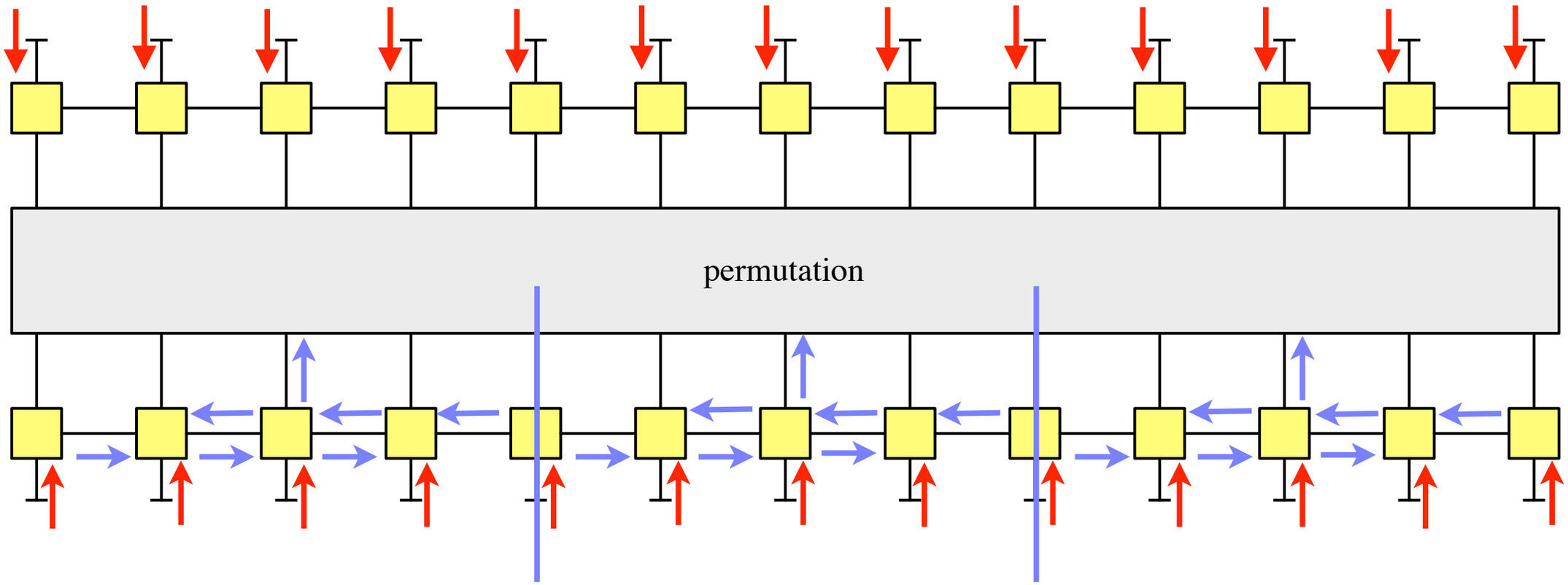
Typical Iterative Schedule



Typical Iterative Schedule



Typical Iterative Schedule



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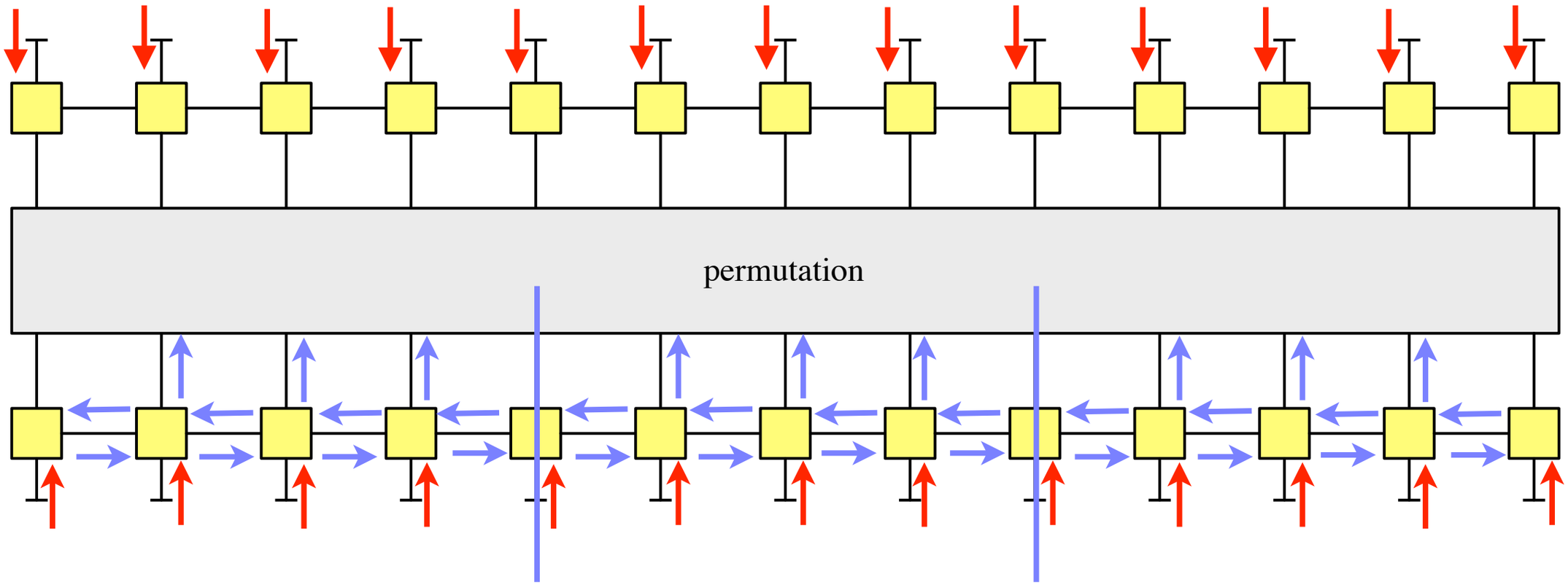

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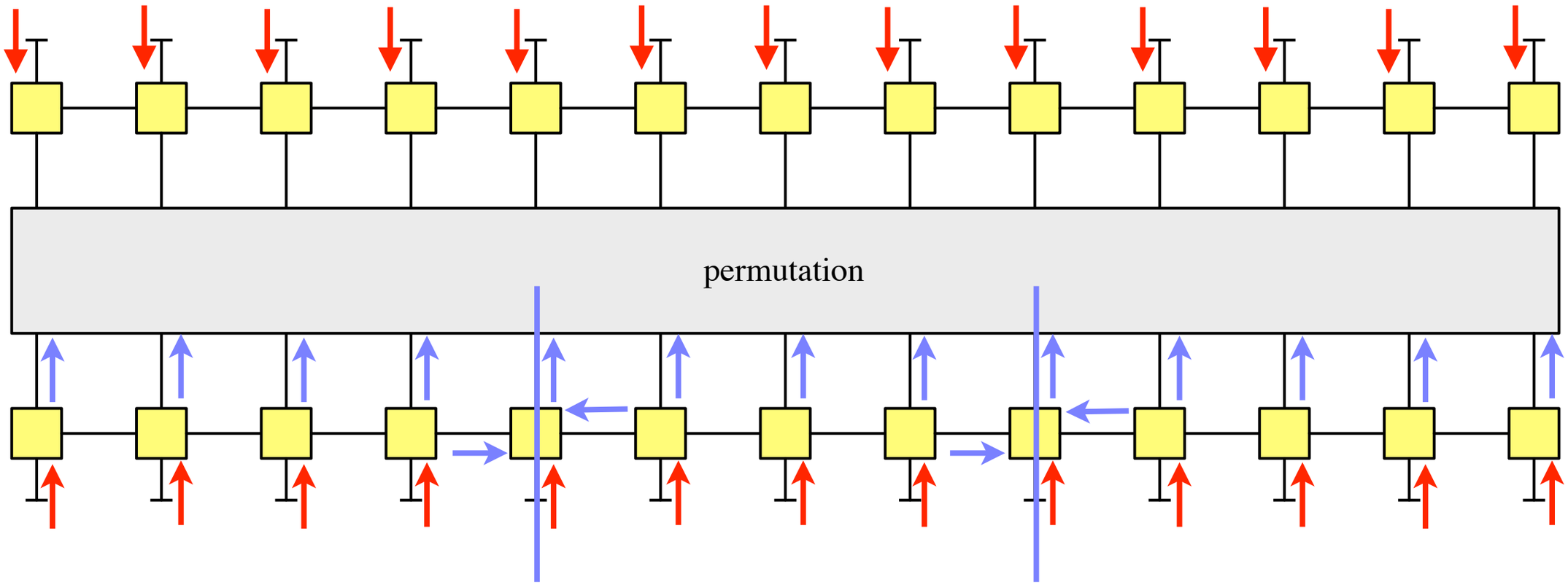

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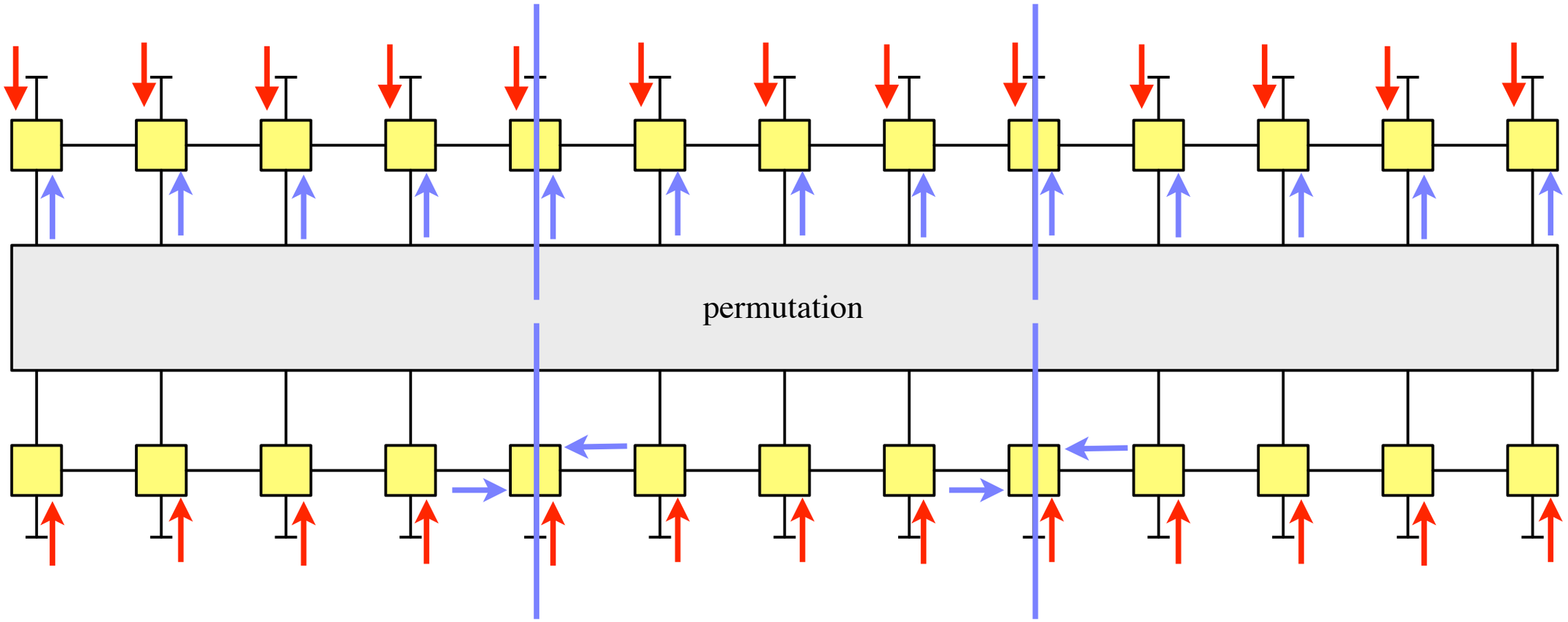
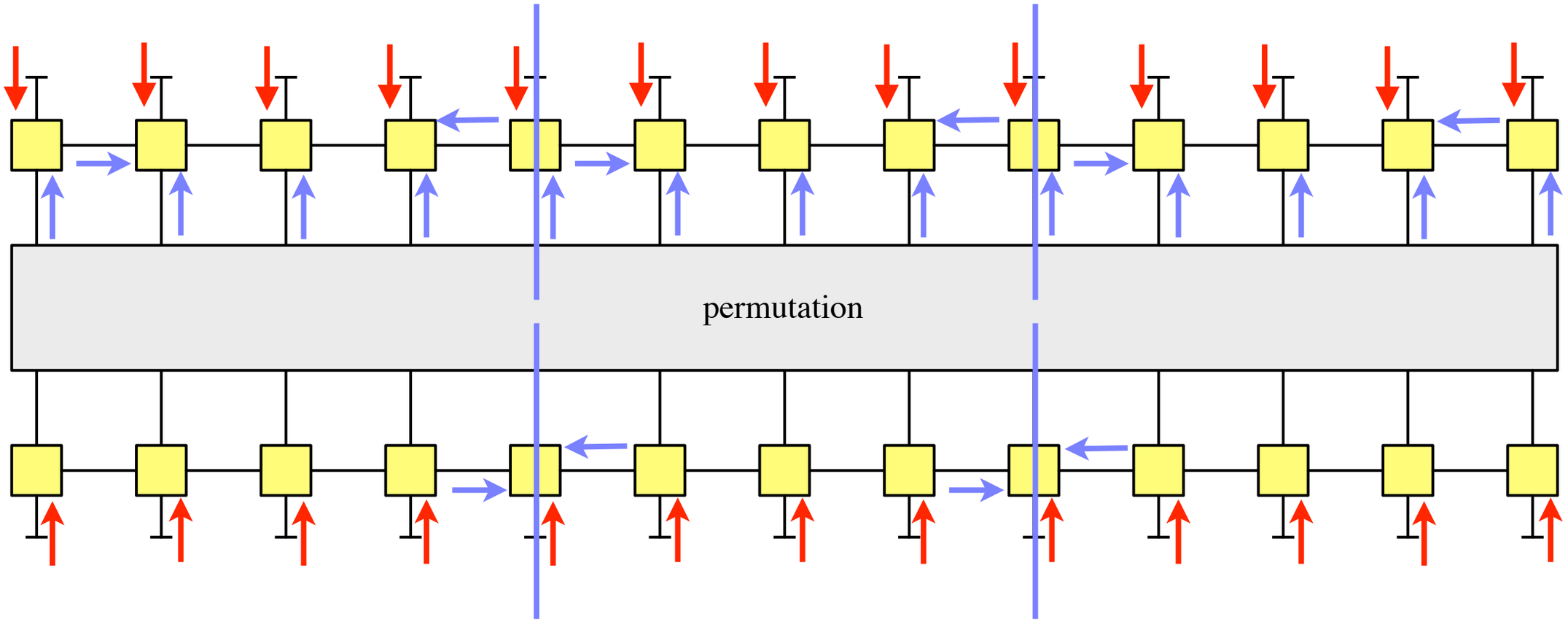


Typical Iterative Schedule



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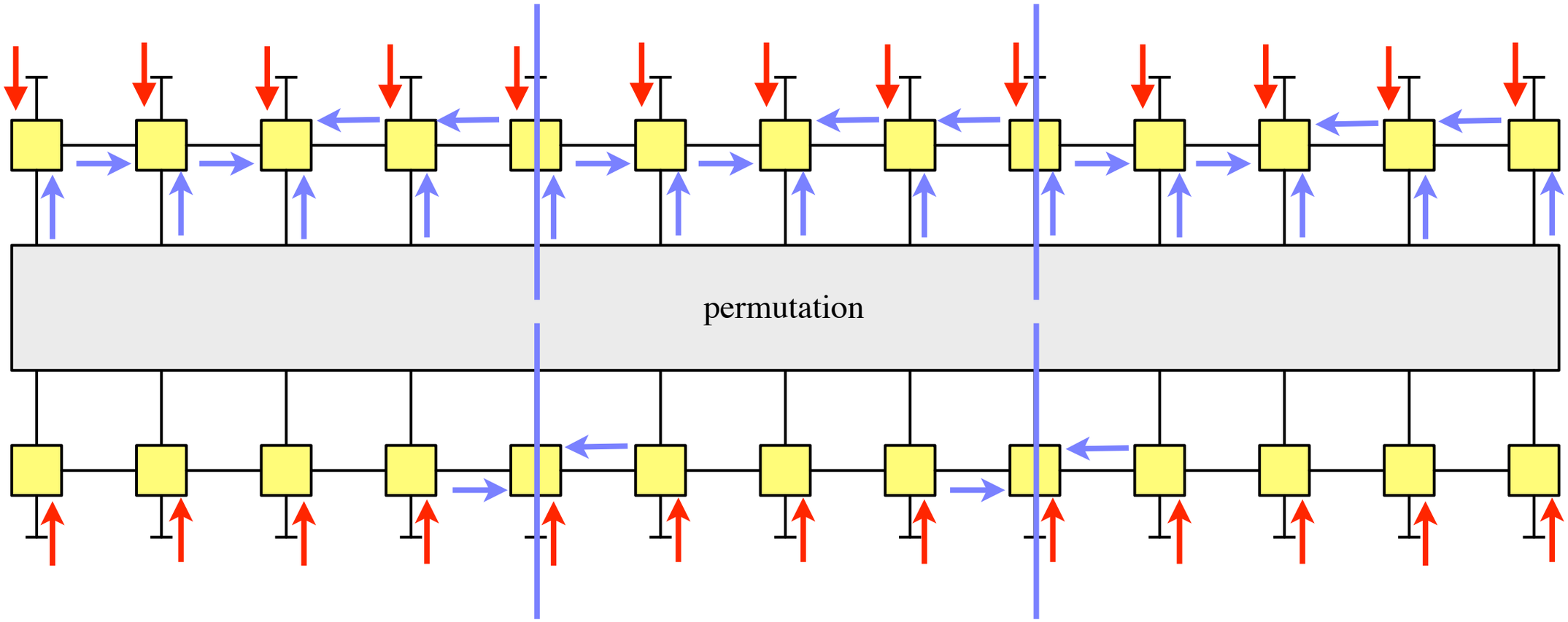
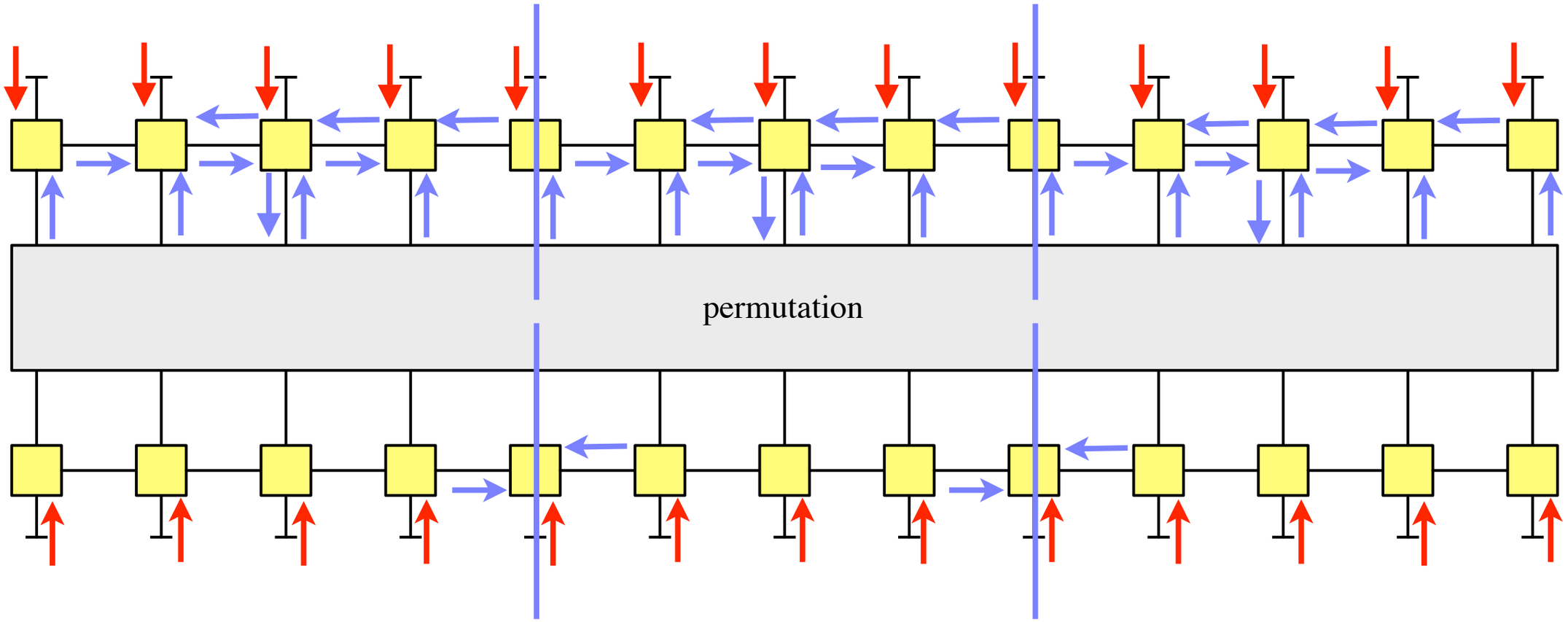
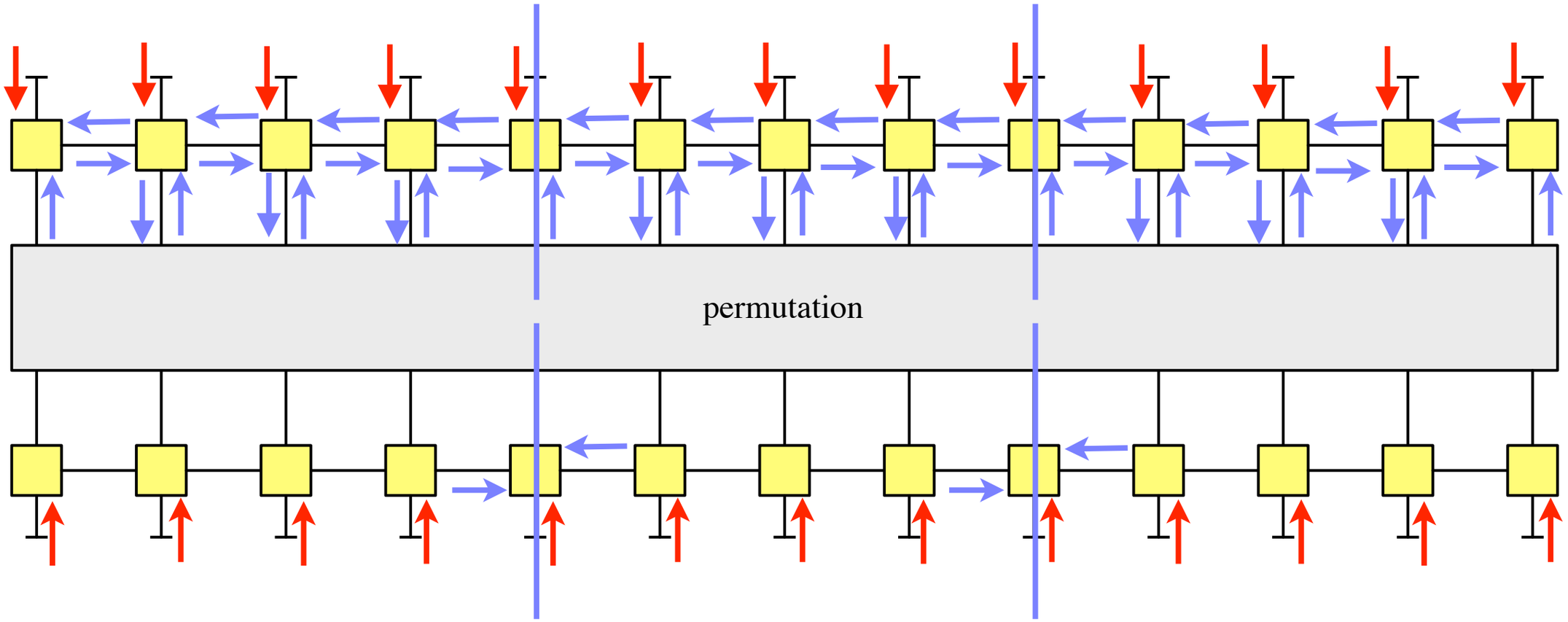


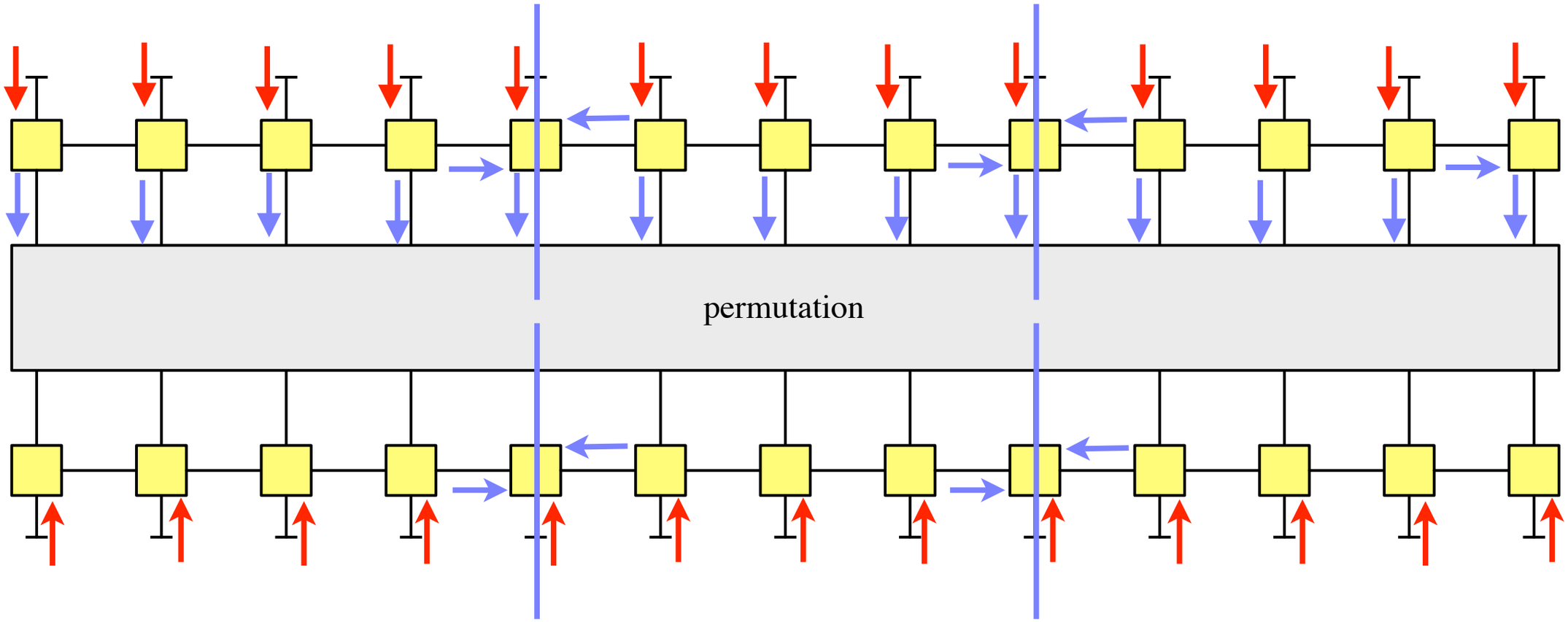
Typical Iterative Schedule



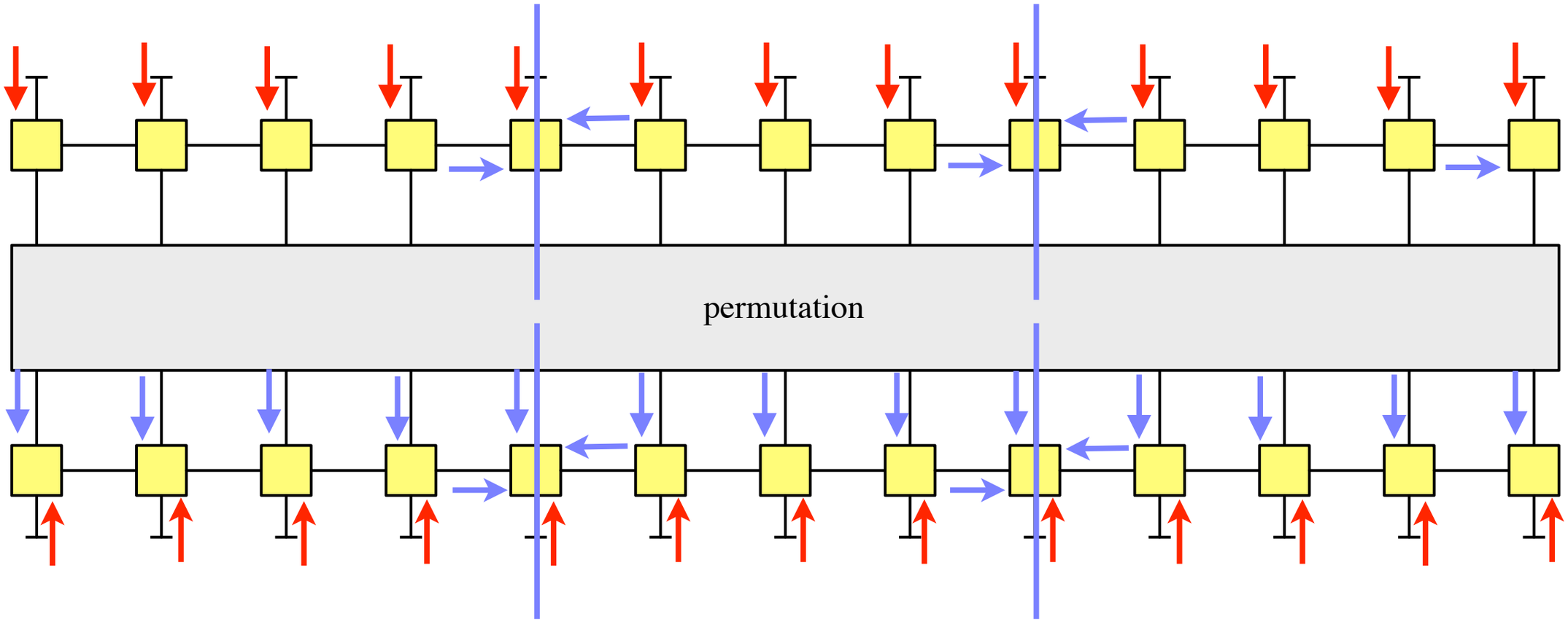
Typical Iterative Schedule



Typical Iterative Schedule



Typical Iterative Schedule



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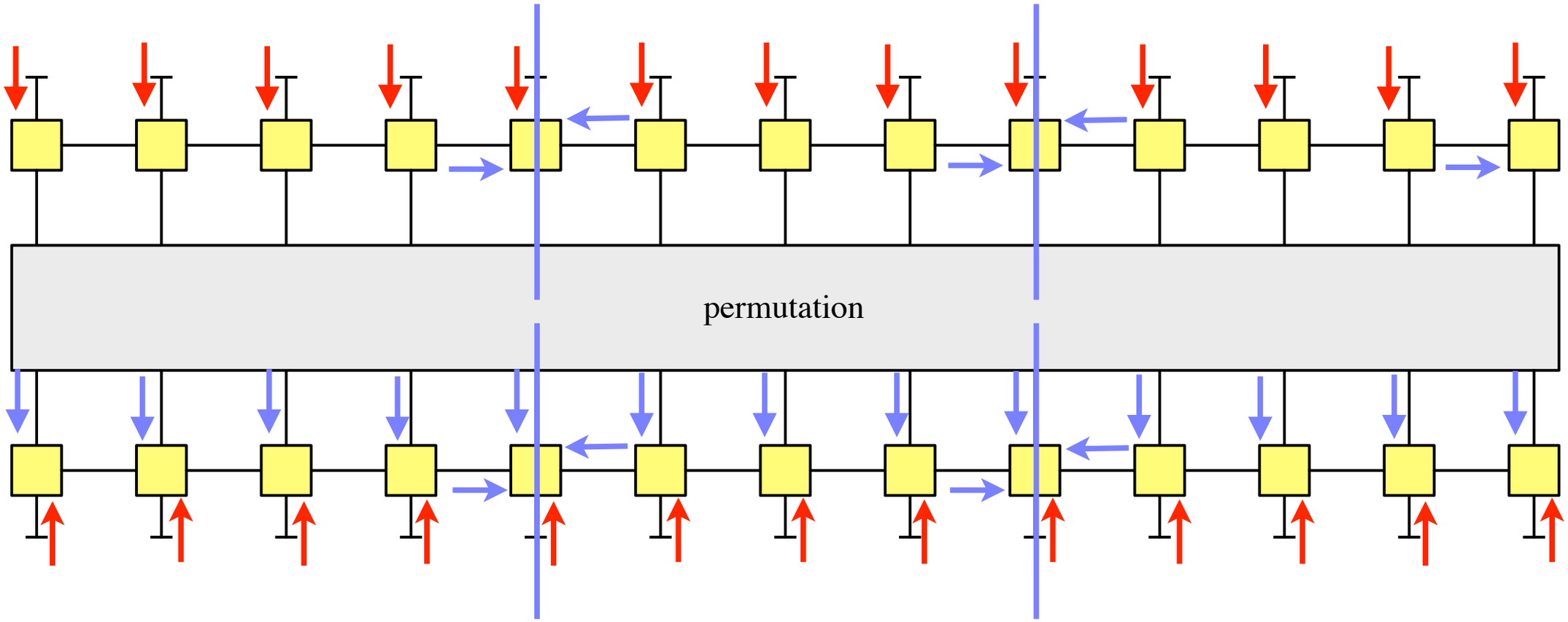

Repeat FBA on each tile with forward and backward messages saved at the tile boundaries