Fading Channels, Diversity and MIMO systems

EE564: Digital Communication and Coding Systems

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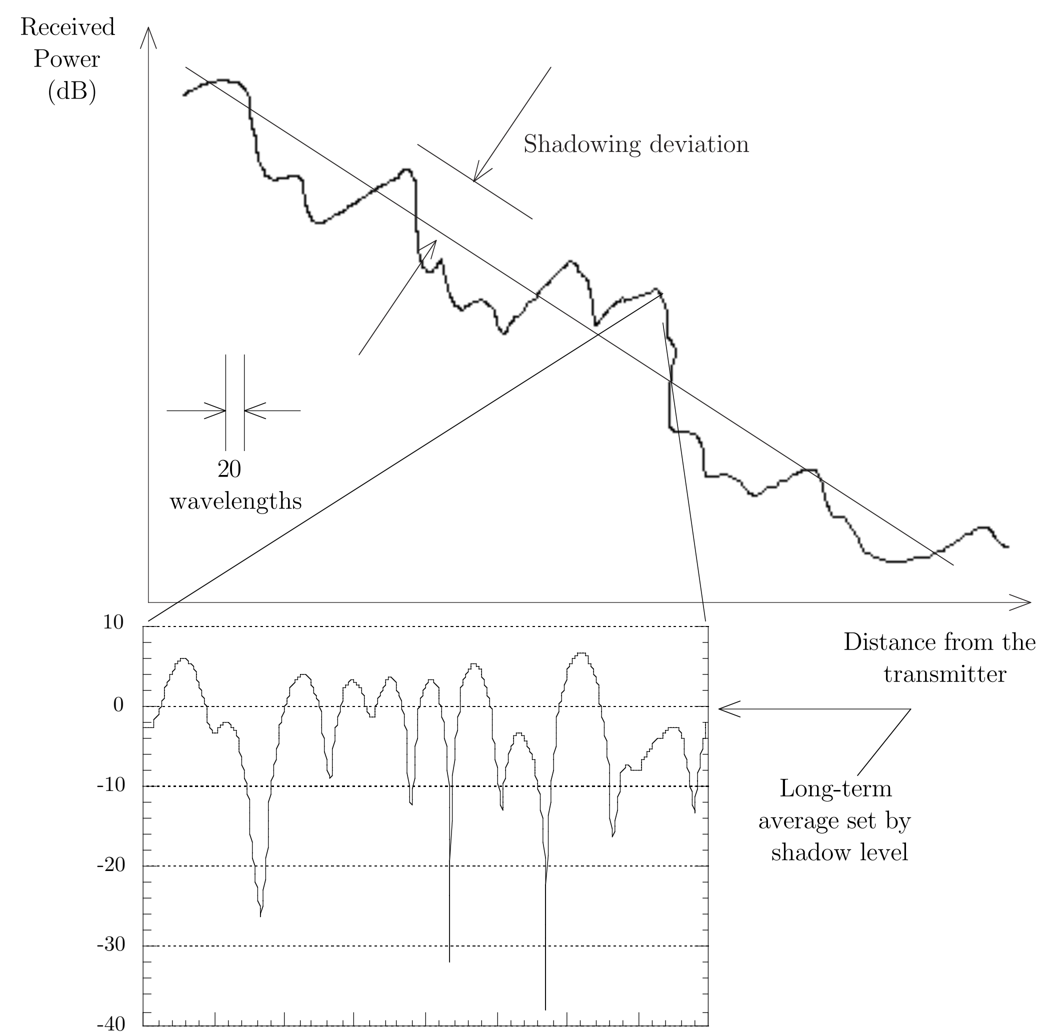
Overview Topics

- Fading channel models
- Performance impact of fading
  - Benefits of diversity
- Methods for obtaining diversity
- MIMO systems
  - Space time codes (for diversity)
  - Space time multiplexing (for increased throughput)
- Capacity measures for MIMO and fading channels

Typical 3-Level Channel Models

- **Path Loss**
  - Deterministic propagation loss model
  - Large scale
  - Empirically determined from field measurements
- **Shadowing**
  - Statistical model for the deviation from the path loss model
  - Long-term fading – *e.g.*, 10-100 wavelengths
  - Empirically determined from field measurements
- **Fading**
  - Statistical model for short-term (sub-wavelength) power fluctuations
  - Also characterizes the distortion characteristics of the channel
  - Simple analytical models, verified via measurements

Relation Between Three Levels of Channel Models



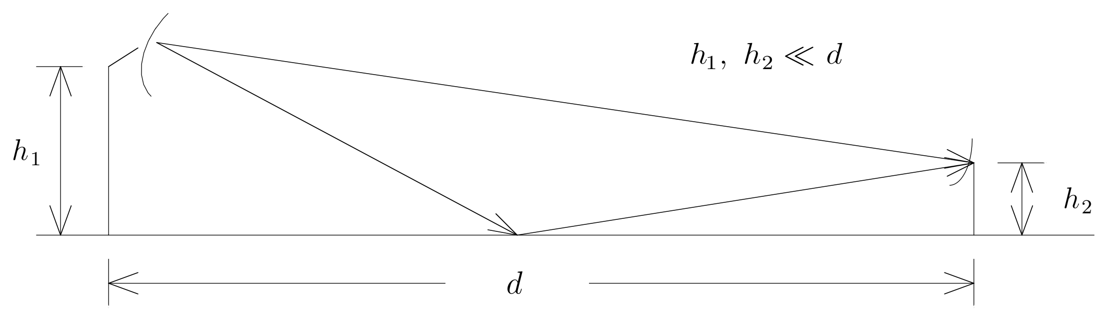
Path Loss Models

- **Free Space:**

$$\frac{P_r(d)}{P_r(d_0)} = \left(\frac{d}{d_0}\right)^{-2}$$

  - Power spread evenly over sphere of radius d

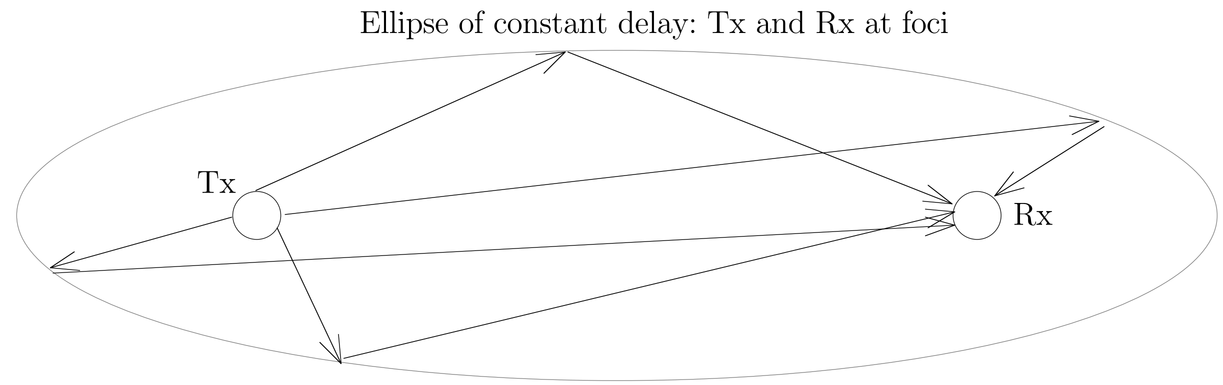
- **Single Ground Reflection:**



$$\frac{P_r(d)}{P_r(d_0)} = \left(\frac{d}{d_0}\right)^{-4}$$

Path Loss Models

- **Multipath Reflection Environments:**



$$\frac{P_r(d)}{P_r(d_0)} = \left(\frac{d}{d_0}\right)^{-\beta}$$

$$\left[\frac{P_r(d)}{P_r(d_0)}\right]_{dB} = -10\beta \log_{10}\left(\frac{d}{d_0}\right)$$

- β is the *path loss exponent*
  - Typical macrocellular: $\beta \sim 3$ to 4
  - Typical microcellular: $\beta \sim 2$ to 8

Path Loss Models

- **Models are Roughly Frequency Independent**
  - Weak dependency described in more detailed model
  - More difficult to predict in smaller regions (*e.g.*, indoor)
  - Environment specific models: ray-tracing, Manhattan pico cells, etc.
- **Power decays linearly (in dB) with delay**
  - Free space $\Rightarrow$ 20 dB per decade
  - $\beta \Rightarrow 10\beta$ dB per decade
- **Utility of path loss models:**
  - rough cell planning (*e.g.*, cell size, reuse factors)

Shadowing Models

- Random deviation from path loss model:

$$\frac{P_{r,S}(d;u)}{P_r(d_0)} = \epsilon(u)\frac{P_r(d)}{P_r(d_0)}$$

$$\left[\frac{P_{r,S}(d;u)}{P_r(d_0)}\right]_{dB} = \left[\frac{P_r(d)}{P_r(d_0)}\right]_{dB} + 10\log_{10}\left[\epsilon(u)\right]$$

$$= -10\beta\log_{10}\left(\frac{d}{d_0}\right) + \epsilon_{dB}(u)$$

- **Common Model:** Log-Normal Shadowing

$$\epsilon_{dB}(u) \sim \mathcal{N}(\cdot; 0; \sigma^2_{\epsilon_{dB}})$$

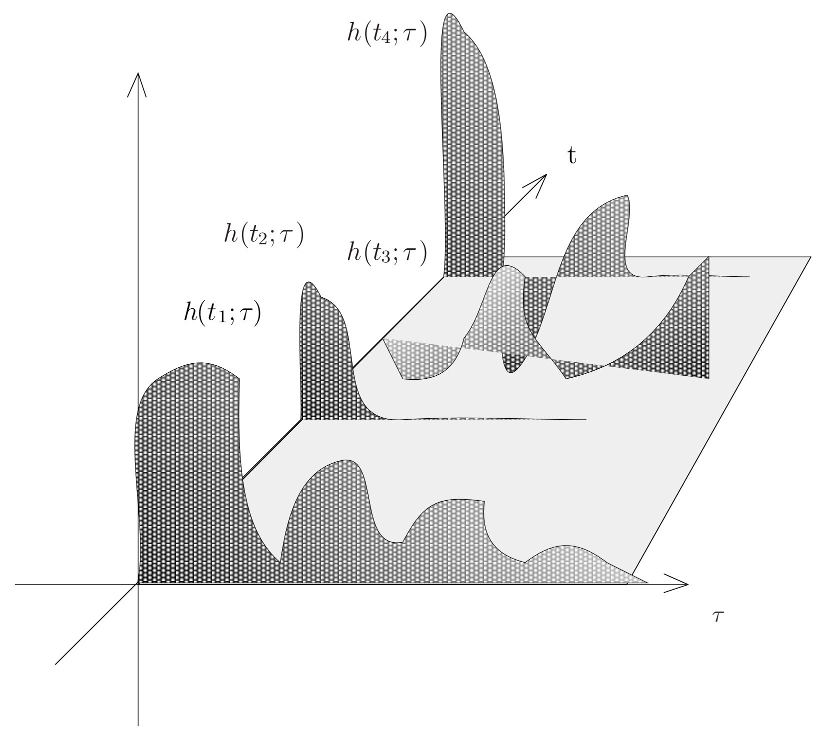
  – The received power in dB may be thought of as Gaussian with mean given by the path loss model and variance $\sigma^2_{\epsilon_{dB}}$

- **Shadowing deviation:** $\sigma_{\epsilon_{dB}}$
  – Macrocellular systems have values in the range 5 to 12, with 8 being typical

Short-term (multipath) Fading Models

- **Common Model:** random, time-varying linear system
  - Impulse response from a delta applied at time t is $h(u;t;\tau)$

$$y(u,\tau) = h(u;t;\tau) * x(\tau) \quad z(u,\tau) = h(u;t+\delta;\tau) * x(\tau)$$
$$z(u,\tau) \neq y(u,\tau)$$



Short-term (multipath) Fading Models

- **Characterizing Distortion:** What is the shape of the impulse response $h(u; t; \tau)$ wrt τ?
  - τ_d: *Delay Spread* – how long does the channel ring from a time impulse?
  - B_c: *Coherence Bandwidth* – over what range of frequencies is the gain of the channel flat?
- **Characterizing Time-variation:** How does $h(u; t; \tau)$ change with t?
  - t_c: *Coherence time* – for what value of Δ are the responses at t and $t + \Delta$ uncorrelated?
  - f_d: *Doppler Spread* – how much will the spectrum of an input tone (*i.e.*, frequency impulse) be spread in frequency?

Short-term (multipath) Fading Models

	Time-variation Properties	Distortion Properties
	variation in t	variation in τ
Time Domain	Coherence Time	Delay Spread
Frequency Domain	Doppler Spread	Coherence Bandwidth

- **Distortion Properties:** $B_c \propto \frac{1}{\tau_d}$
- **Time-variation Properties:** $f_d \propto \frac{1}{t_c}$

Measures Relative to Signals

- **Does the channel distort the signal?**
  - $W \ll B_c \Rightarrow$ NO $\Rightarrow$ *Flat Fading*
  - $W \geq B_c \Rightarrow$ YES $\Rightarrow$ *Frequency-Selective Fading*
    - **Note:** If $W \cong \frac{1}{T}$, then frequency selective fading implies that $T \leq \tau_d \Rightarrow$ time dispersion or *intersymbol interference (ISI)*
    - Not so for wideband systems – $W \gg \frac{1}{T}$
    - Flat Fading $\iff$ amplitude and phase distortion only!
- **Does the channel remain constant over many channel uses?**
  - $T \ll t_c \Rightarrow$ YES $\Rightarrow$ *Slow Fading*
  - $T \geq t_c \Rightarrow$ NO $\Rightarrow$ *Fast Fading*
    - Slow fading may still require frequent training and/or adaptive tracking

Clarke's Doppler Model: Meaning (flat fading)

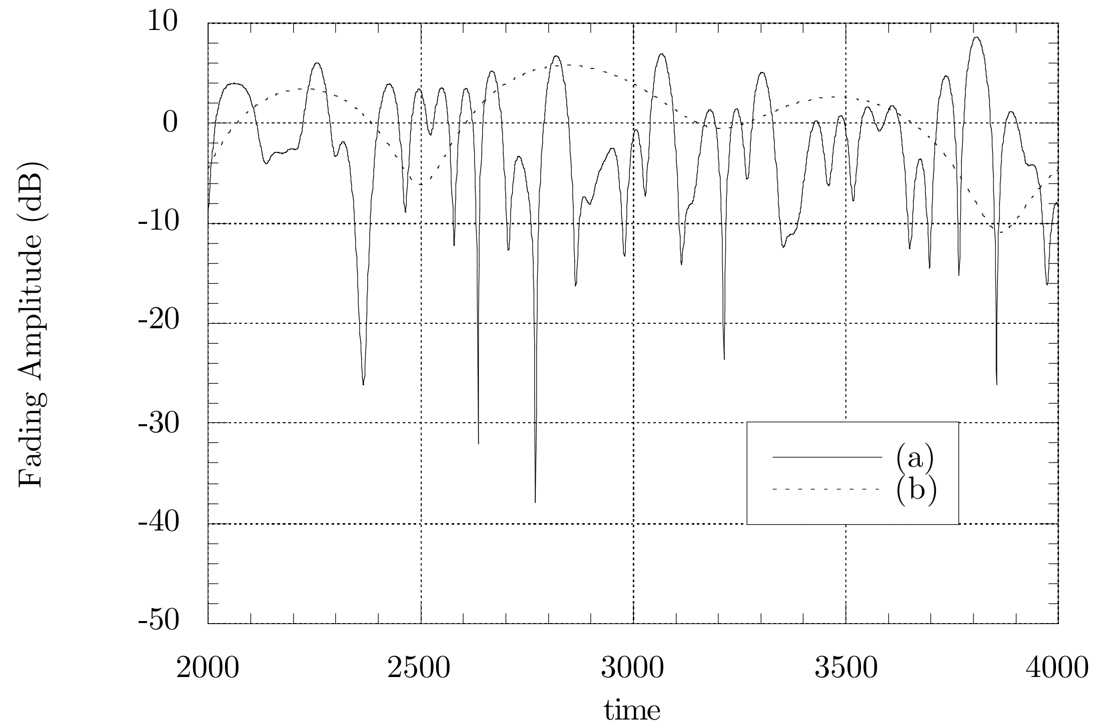
- **I/Q carrier modulated inputs:**

$$
\begin{aligned}
x(t) &= x_I(t)\sqrt{2}\cos(2\pi f_c t) - x_Q(t)\sqrt{2}\sin(2\pi f_c t) \\
&= \Re\left\{\bar{x}(t)\sqrt{2}e^{j2\pi f_c t}\right\} \\
&= |\bar{x}(t)|\cos(2\pi f_c t + \angle\bar{x}(t)) \\
\bar{x}(t) &= x_I(t) + j x_Q(t)
\end{aligned}
$$

- **Output:**

$$
\begin{aligned}
y(u;t) &= [h_I(t)x_I(t) - h_Q(t)x_Q(t)]\sqrt{2}\cos(2\pi f_c t) \\
&\quad -[h_I(t)x_Q(t) + h_Q(t)x_I(t)]\sqrt{2}\sin(2\pi f_c t) \\
&= \Re\left\{\bar{y}(t)\sqrt{2}e^{j2\pi f_c t}\right\} \\
&= |\bar{y}(t)|\cos(2\pi f_c t + \angle\bar{y}(t)) \\
\bar{y}(t) &= y_I(t) + j y_Q(t) = \bar{x}(t)\bar{h}(t)
\end{aligned}
$$

Power in Sample Realizations



this is the envelope for Rayleigh (flat) fading

Fading Channel Summary

- In general, this is complex stuff…
- Many modern systems use OFDM, so the sub-carrier channels are modeled as frequency flat fading.
  - Correlation in complex gains across frequencies, several coherence bandwidth in a broadband OFDM system
  - Rayleigh fading is worse case: I and Q channel gains are zero mean, independent Gaussian. Results from many, many diffuse scatters
  - Ricean fading is similar with non-zero means in the I and Q channel gains
- Time variation is often modeled as
  - Fixed or quasi-static

Effects of Fading

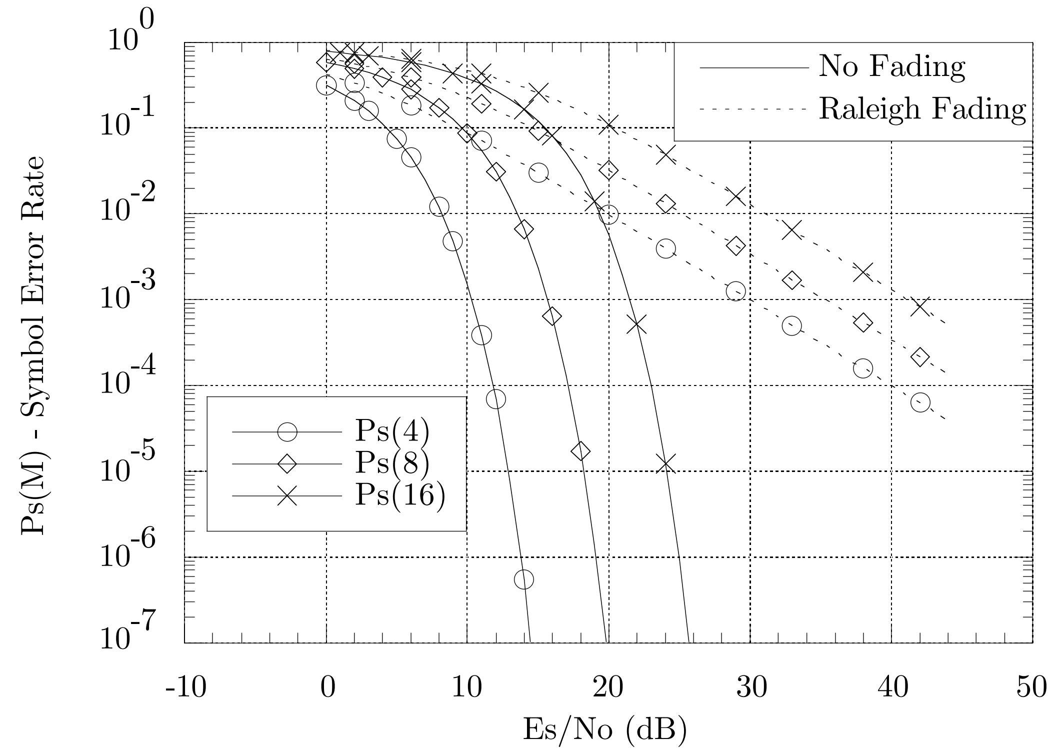
- **Recall:** for the AWGN channel, for all modulations considered, the error performance decays exponentially in SNR

$$P_b \cong K_1 e^{-K_2 \frac{E_b}{N_0}}$$

- **Fading:**
  - Random variations in received power
  - Average the AWGN performance over the statistics E_b/N_0
  - Consider the performance as a function of average E_b/N_0
  - Performance decays only inverse linearly with Rayleigh (flat) fading

$$P_b \cong K \left[\frac{E_b}{N_0}\right]^{-1}$$

Effects of Fading – PSK



- **Intuition:** worst case dominates!

$$\alpha 10^{-1} + (1-\alpha)10^{-6} \cong \alpha 10^{-1} \gg 10^{-6}$$

Combating Fading: Diversity

- **Intuition:** combining multiple independent copies of the received signal will reduce the *variance* of the SNR

$$\bar{r}^{(d)}(t) = \bar{h}^{(d)} s(t; \mathbf{a}) + \bar{n}^{(d)}(t) \quad d = 1, 2 \ldots D$$

  – *Diversity Order: D* – number of effectively independent replicas
  – *Impact on Performance:* Increases BER decay

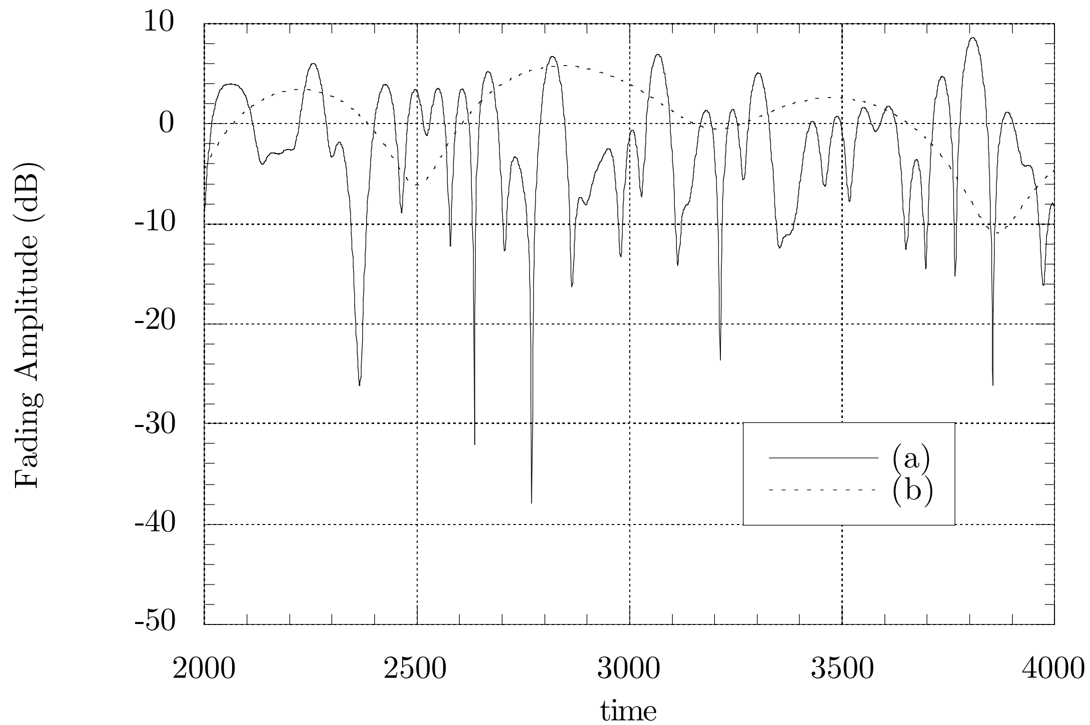
$$P_b \cong K \left[\frac{E_b}{N_0}\right]^{-D}$$

  – As D increases, the performance approaches that of no-fading!

How to Obtain Diversity

- **Spatial Diversity:**
  - *e.g.*, Space two antennas farther than $\lambda/2$ in dense scattering
- **Time Diversity:**
  - *e.g.*, Repeat the transmission after waiting longer than the coherence time
- **Frequency Diversity:**
  - *e.g.*, Transmit the signal on two carriers spaced further than the coherence BW
- Which type if best?
  - Performance gains are the same regardless (nominally)
  - Effort required to combine the diversity effectively may differ greatly with the type and the exact signal format

Intuitive View of Diversity



Optimal Diversity Combining

- **Optimal Digital Communication Receiver:**
  - Consider all possible versions of the received signal (including distortion, interference, etc.) that arise from possible $\mathbf{a}$
  - Correlate with each of these possibilities
  - Adjust correlation for energy difference
  - Maximize over possibilities
- This yields **Maximum (Signal-to-Noise) Ratio Combining:**

$$z_d(\tilde{\mathbf{a}}) = \int \bar{r}^{(d)}(t) s(t; \tilde{\mathbf{a}}) dt$$

$$Z(\tilde{\mathbf{a}}) = \sum_{d=1}^{D} \left(\bar{h}^{(d)}\right)^* z_d(\tilde{\mathbf{a}})$$

  - If each signal $s(t; \tilde{\mathbf{a}})$ has equal energy, then

$$\max_{\tilde{\mathbf{a}}} Z(\tilde{\mathbf{a}})$$

Performance of BPSK in Rayleigh Fading

$$\gamma = E_b/N_0 = \text{random due to fading with mean } \overline{\gamma} = \overline{E}_b/N_0$$

$$f(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}} \quad \gamma > 0$$

Rayleigh fading (no diversity)

$$P(\mathcal{E}) = \int_0^\infty \mathrm{Q}(\sqrt{2\gamma}) f(\gamma) d\gamma$$

$$= \frac{1}{2}\left[1 - \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right]$$

$$\approx \frac{1}{4\overline{\gamma}} \qquad \overline{\gamma} \gg 1$$

Note: all mods we have seen have uncoded performance that is well approximated as a Q-function

Performance of BPSK in Rayleigh Fading

$$\gamma = E_b/N_0 = \text{random due to fading with mean } \overline{\gamma} = \overline{E}_b/N_0$$

$$f(\gamma) = \frac{1}{(D-1)!\overline{\gamma}^D}\gamma^{D-1}e^{-\gamma/\overline{\gamma}} \qquad \gamma > 0$$

Rayleigh fading, diversity D and MRC combining

Central chi-squared with 2D degrees of freedom

$$P(\mathcal{E}) = \int_0^\infty \mathrm{Q}(\sqrt{2\gamma})f(\gamma)d\gamma$$

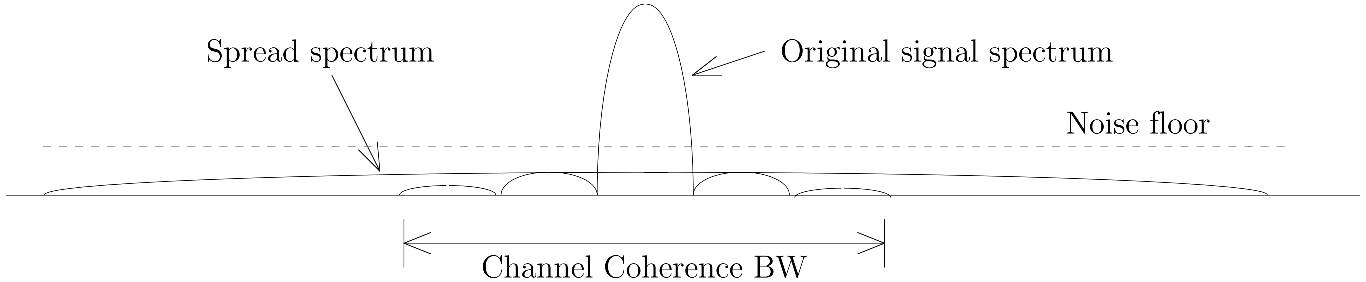
$$= \left(\frac{1-\mu}{2}\right)^D \prod_{k=0}^{D-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k$$

$$\mu = \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}$$

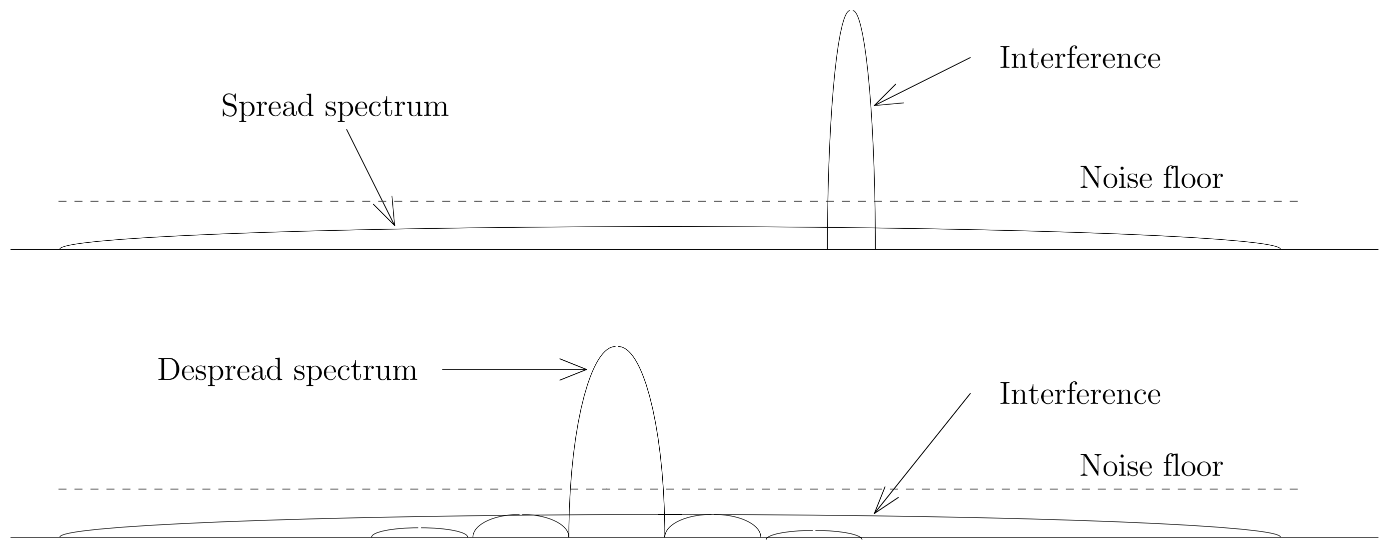
$$P(\mathcal{E}) \approx \binom{2D-1}{D} \left(\frac{1}{4\overline{\gamma}}\right)^D \qquad \overline{\gamma} \gg 1$$

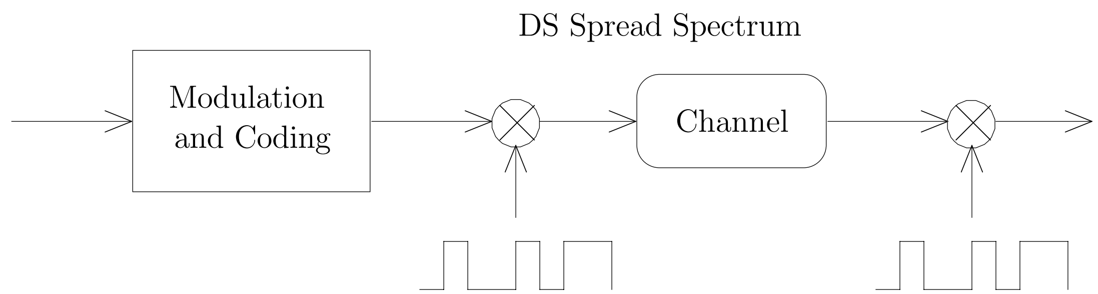
Practical Frequency Diversity: Spreading

- **Use more bandwidth than required:**
  - provides frequency diversity $\Longleftrightarrow$ frequency-selectivity
  - spectrally inefficient (single-user)
- **Techniques:**
  - *Direct Sequence:* mix with a pseudorandom squarewave carrier
  - *Frequency Hopping:* change f_c according to a pseudorandom pattern
  - *Time Hopping:* change signal epoch of narrow pulse in pseudorandom manner



DS Spread Spectrum

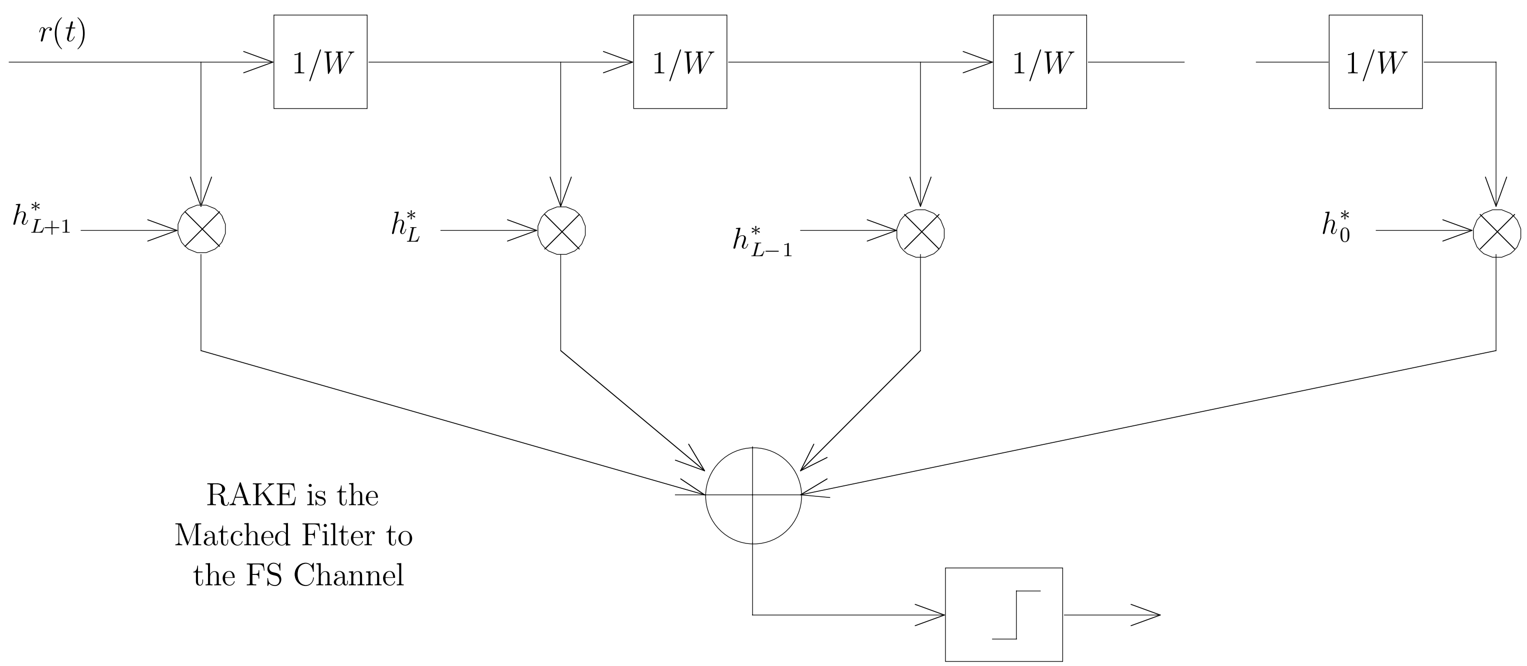




D ~ number of coherence BWs in the spread BW

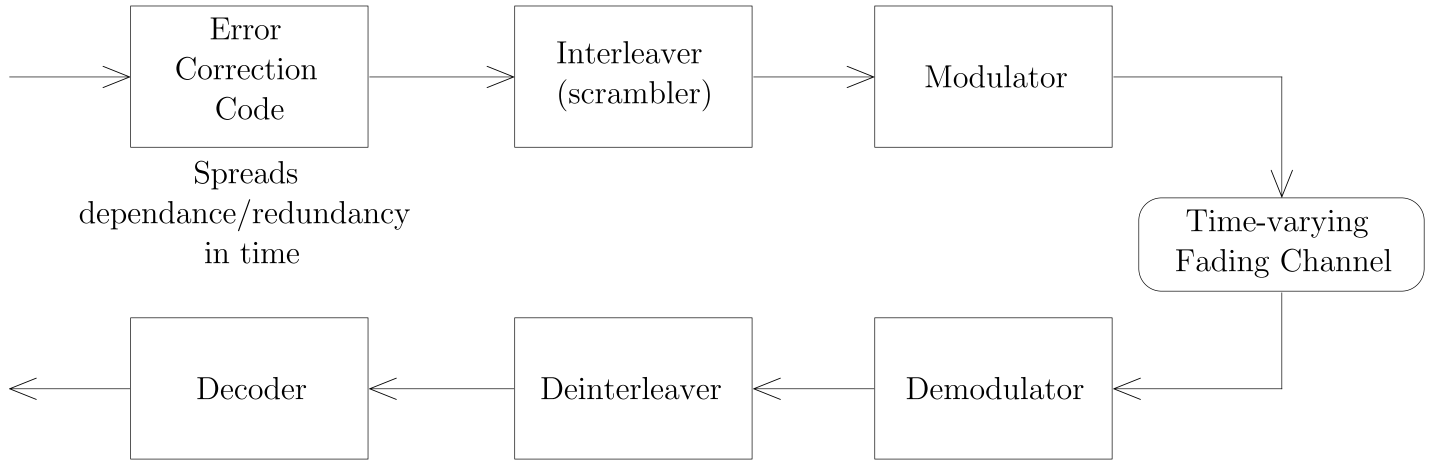
DS Spread Spectrum

- **Spreading Ratio:** $\eta = T_b/T_c$; T_c = chip time
  - Also called *processing gain* since an interferer's in-band power is reduce by η^{-1} after despreading
- **Frequency Diversity Combining:** RAKE receiver



Practical Time Diversity: Interleaving and Coding

- **Forward Error Correction Coding:**
  - Provides an SNR gain (*i.e.*, coding gain) on AWGN channel
  - Also provides (small) diversity gain on a time-varying fading channel
- **Interleaving:**
  - Greatly improves the diversity gain associated with coding
  - Useless without coding

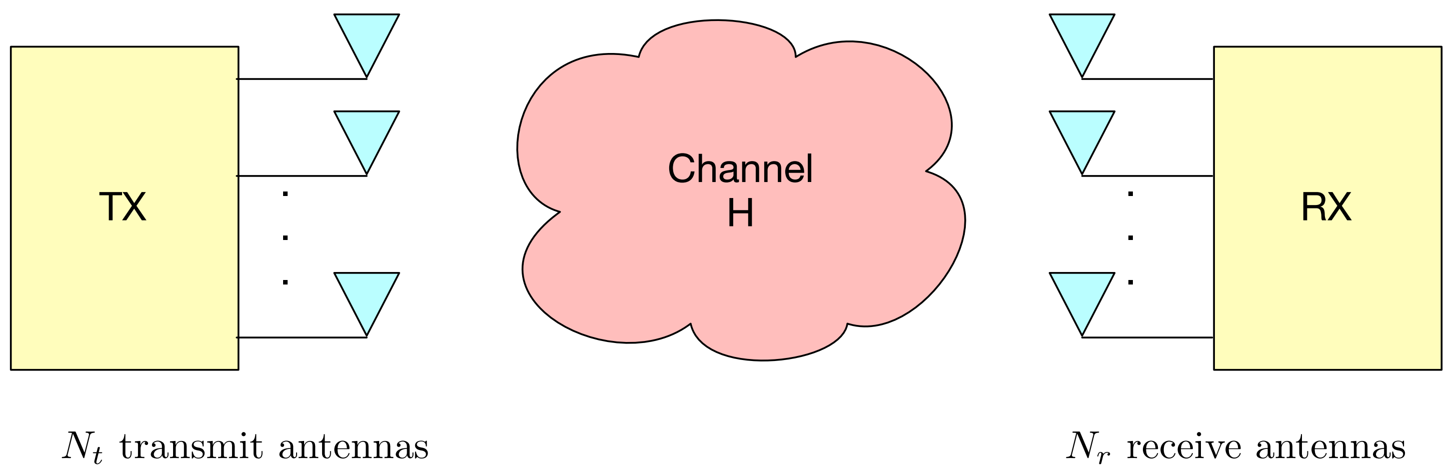


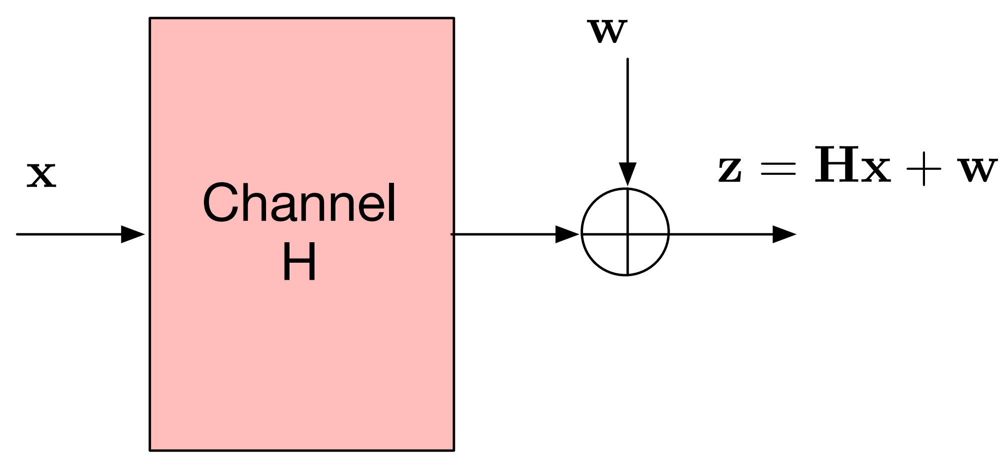
D ~ number of coherence times in the code block

Practical Diversity

- In the above, we do not have access to parallel, decoupled diversity branches
  - diversity is coupled together through the signaling
    - general results still hold
      - obtained by doing some form of whitening/decorrelation on the correlated fading metrics

MIMO Systems





This is a single channel use

MIMO Systems

- **Typical Channel Model**
  - Each element of **H** is an independent, flat-fading, Rayleigh channel
- **Space Time Codes (STCs):**
  - Use to get diversity against multi path fading
    - Typically model the channel as not changing during code blocks
    - Very short code blocks — these are really ST Modulations
- **Space Time Multiplexing:**
  - Just send a different QASK signal over each TX antenna
    - If Nt >= Nr, can support Nt "spatial streams"

Space Time Codes

744 IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 44, NO. 2, MARCH 1998

Space–Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction

Vahid Tarokh, *Member, IEEE*, Nambi Seshadri, *Senior Member, IEEE*, and A. R. Calderbank, *Fellow, IEEE*

Suggest basic design rules for STCs: Rank and Determinant Criterion

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 47, NO. 4, APRIL 1999 527

Signal Design for Transmitter Diversity Wireless Communication Systems Over Rayleigh Fading Channels

Jiann-Ching Guey, Michael P. Fitz, Mark R. Bell, and Wen-Yi Kuo, *Member, IEEE*

IEEE JOURNAL ON SELECT AREAS IN COMMUNICATIONS, VOL. 16, NO. 8, OCTOBER 1998 1451

A Simple Transmit Diversity Technique for Wireless Communications

Siavash M. Alamouti

Very simple STC code for Nt = 2

Space Time (block) Codes

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{w}_k \qquad k = 0, 1, 2, \ldots L-1$$

$$\mathbf{Z} = \mathbf{H}\mathbf{X} + \mathbf{W}$$

For receiver with CSI, the ML receiver is

$$\min_{\mathbf{x} \in \mathcal{C}} \|\mathbf{Z} - \mathbf{H}\mathbf{X}\|^2$$

The pairwise error probability, conditioned on **H**, will be:

$$P_{PW}(\mathbf{X}^i, \mathbf{X}^j | \mathbf{H}) = \mathrm{Q}\left(\sqrt{\frac{d^2(i,j)}{2N_0}}\right)$$

need to average over the fading

$$d^2(i,j) = \|\mathbf{H}(\mathbf{X}^i - \mathbf{X}^j)\|^2$$

Space Time (block) Codes

Remarks on Space-Time Codes Including a New Lower Bound and an Improved Code

Hsiao-feng Lu, *Student Member, IEEE*,
Yuankai Wang, *Student Member, IEEE*, P. Vijay Kumar, *Fellow, IEEE*,
and Keith M. Chugg, *Member, IEEE*

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy$$
$$= \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta$$

Craig form of Q-function

$$P\left(S_1 \to S_2 | H\right) = Q\left(\frac{d_E}{\sqrt{2}}\right)$$

$$d_E^2 = \rho_t \sum_{q=1}^{Q} \lambda_q \sum_{p=1}^{P} |d_{pq}|^2$$

where $\{\lambda_q\}$ are the eigenvalues of $\triangle S \triangle S^\dagger$, where $\triangle S = (S_1 - S_2)$ is the difference-signal matrix, U is the corresponding eigenvector matrix, and $d_{pq} = (HU)_{pq}$. The authors of [2] also make use of the Cher-

same diversity/fading equation as before, but now D and the branch powers depend on signal differences

$$\frac{\kappa}{2} f^{-P}(\eta) < P(S_1 \to S_2) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} f^{-P}\left(\frac{\eta}{\sin^2\theta}\right) d\theta$$
$$\leq \frac{1}{2} f^{-P}(\eta).$$

$$f(x) := \prod_q (1 + \lambda_q x) = \sum_{i=0}^{Q} \sigma_i x^i.$$

Space Time (block) Codes

Design Criteria for Rayleigh Space–Time Codes:

- *The Rank Criterion:* In order to achieve the maximum diversity mn, the matrix $B(\boldsymbol{c}, \boldsymbol{e})$ has to be full rank for any codewords $\boldsymbol{c}$ and $\boldsymbol{e}$. If $B(\boldsymbol{c}, \boldsymbol{e})$ has minimum rank r over the set of two tuples of distinct codewords, then a diversity of rm is achieved. This criterion was also derived in [15].

These are similar to fading channel code design metrics for single-input/single-output channels

- *The Determinant Criterion:* Suppose that a diversity benefit of rm is our target. The minimum of rth roots of the sum of determinants of all $r \times r$ principal cofactors of $A(\boldsymbol{c}, \boldsymbol{e}) = B(\boldsymbol{c}, \boldsymbol{e})B^*(\boldsymbol{c}, \boldsymbol{e})$ taken over all pairs of distinct codewords $\boldsymbol{e}$ and $\boldsymbol{c}$ corresponds to the coding advantage, where r is the rank of $A(\boldsymbol{c}, \boldsymbol{e})$. Special attention in the design must be paid to this quantity for any codewords $\boldsymbol{e}$ and $\boldsymbol{c}$. The design target is making this sum as large as possible. If a diversity of nm is the design target, then the minimum of the determinant of $A(\boldsymbol{c}, \boldsymbol{e})$ taken over all pairs of distinct codewords $\boldsymbol{e}$ and $\boldsymbol{c}$ must be maximized.

Ortogonal STCs

$$\mathbf{X} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$$

Full diversity, full rate STC with very simple decoding

ALAMOUTI: SIMPLE TRANSMIT DIVERSITY TECHNIQUE FOR WIRELESS COMMUNICATIONS 1455

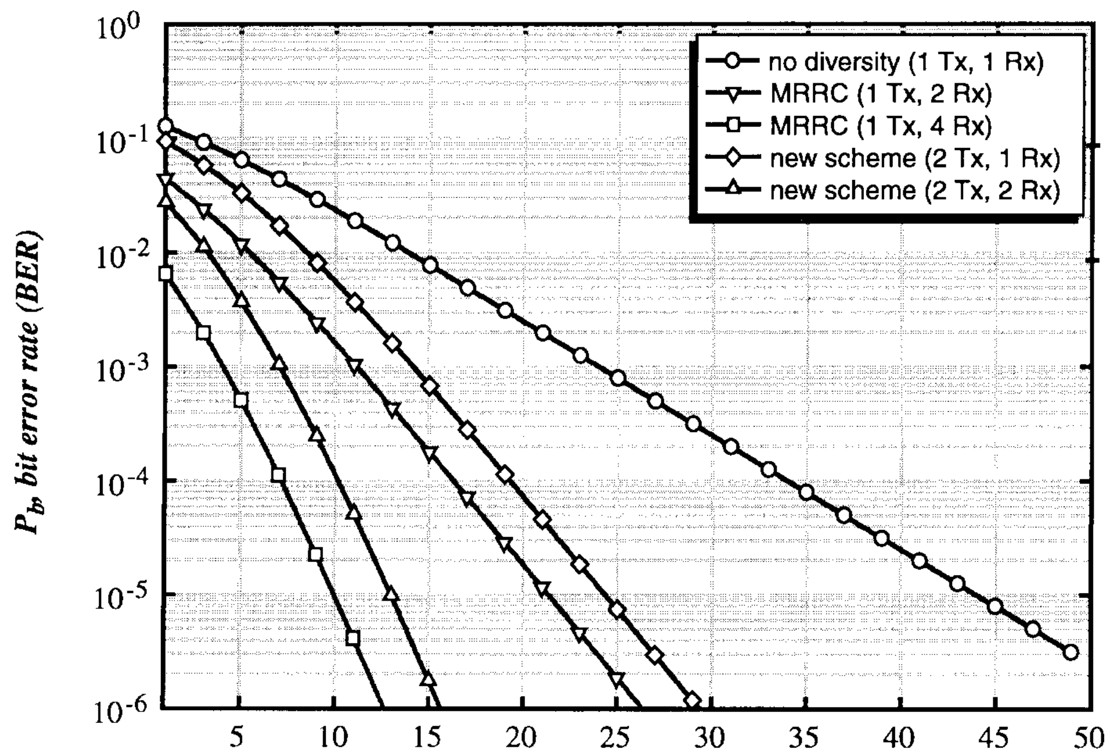


Fig. 4. The BER performance comparison of coherent BPSK with MRRC and two-branch transmit diversity in Rayleigh fading.

Space Time Multiplexing

In STC development, the best one targets is "full rate (rate 1)" — i.e., if the channel is used L times with M-ary constellation, then there should be M*L STC code matrices

In ST-MUX, we send an M-ary signal point out of each TX antenna at each time — these are "rate Nt" under the STC rate definition

3306 IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 6, NO. 9, SEPTEMBER 2007

Capacity for Suboptimal Receivers for Coded Multiple-Input Multiple-Output Systems

Pansop Kim and Keith M. Chugg, *Member, IEEE*

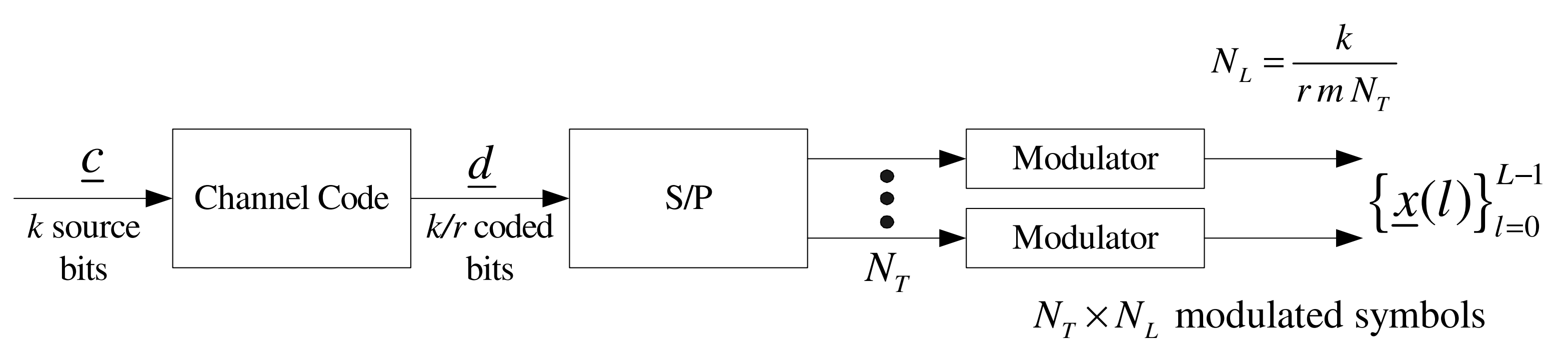


Fig. 1. Coded multiple-input multiple-output transmitter and suboptimal receiver.

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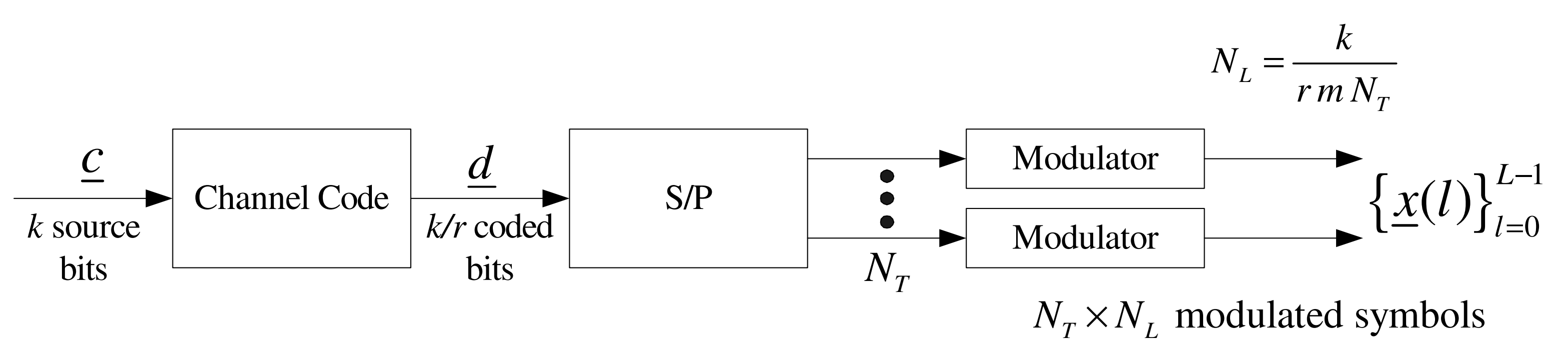
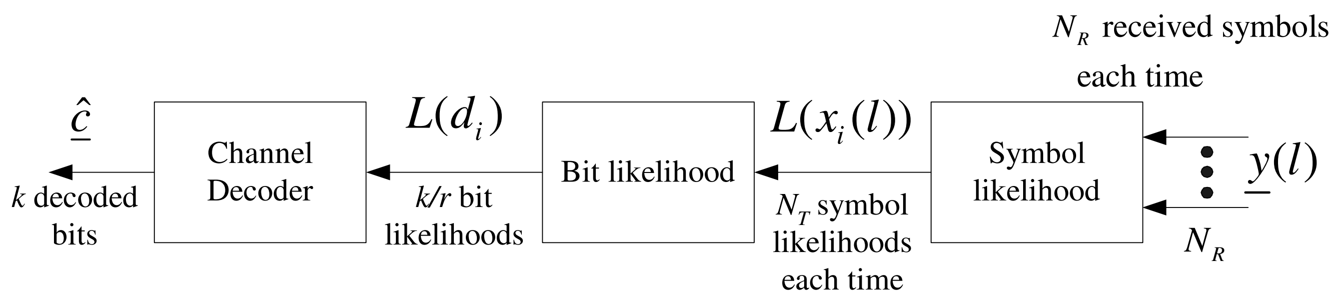


Fig. 1. Coded multiple-input multiple-output transmitter and suboptimal receiver.

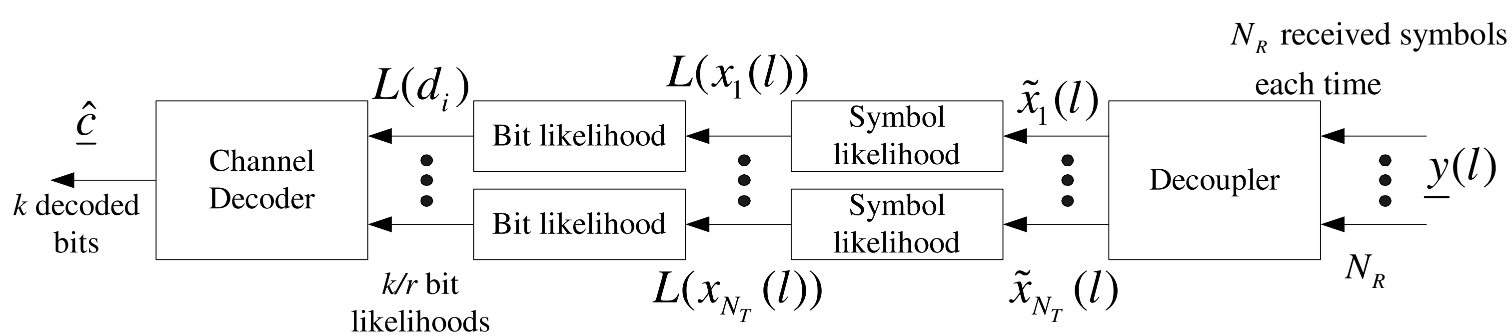
Space Time Multiplexing

$$\underline{y}(l) = \mathbf{H}(l)\underline{x}(l) + \sqrt{\frac{N_T}{\rho}}\underline{n}(l)$$



(a)

this is SO-demod



(b)

sub-optimal, linear (stream) decoupler

Linear Minimum Mean Square Error (LMMSE) decoupler is the most widely used

Space Time Multiplexing

$$\underline{y}(l) = \mathbf{H}(l)\underline{x}(l) + \sqrt{\frac{N_T}{\rho}}\underline{n}(l)$$

A. LZF, LMMSE decouplers

Let $\mathbf{A}^H$ be a spatial linear filter, then the decoupled vector, $\underline{\tilde{x}}$ becomes

$$\underline{\tilde{x}} = \mathbf{A}^H \underline{y} \tag{3}$$

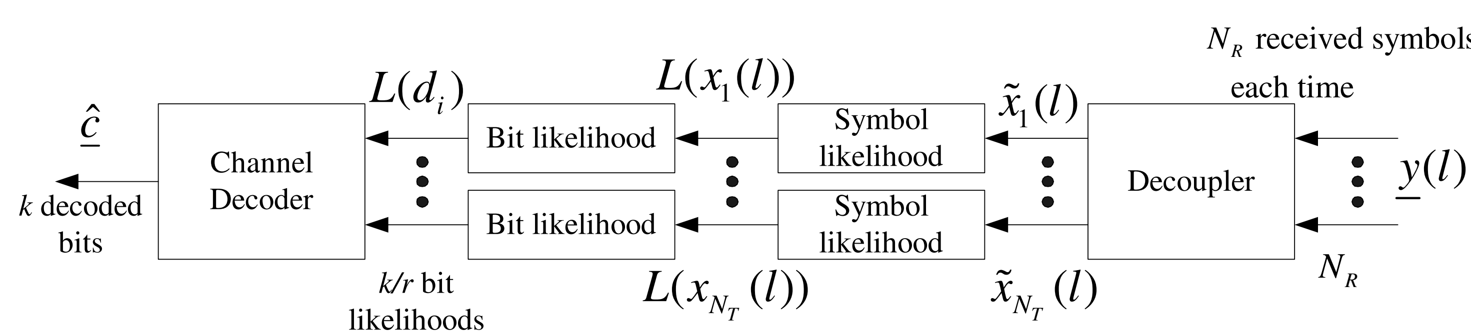
where $(\cdot)^H$ is the complex conjugate and transpose of a vector or matrix. For the LZF decoupler, $\mathbf{A}^H$ is a pseudo-inverse of $\mathbf{H}$, i.e.,

$$\mathbf{A}^H = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H, \tag{4}$$

Note that $\mathbf{H}^H\mathbf{H}$ is invertible with probability one[1] if N_R is not smaller than N_T. For the LMMSE decoupler, $\mathbf{A}^H$ minimizes the mean-squared error, which is $\mathbb{E}\{|\underline{x} - \mathbf{A}^H\underline{y}|^2\}$. Then,

$$\mathbf{A}^H = \left(\mathbf{H}^H\mathbf{H} + \frac{N_T}{\rho}\mathbf{I}\right)^{-1}\mathbf{H}^H. \tag{5}$$

Linear Minimum Mean Square Error (LMMSE) decoupler is the most widely used



MIMO Capacity Measures

$$C(\mathbf{H}) = \log_2\left(\det\left[\mathbf{I} + (\rho/N_t)\mathbf{H}\mathbf{H}^\dagger\right]\right)$$

$$C_{\text{out}}(p) : \text{P}\textsc{r}\left\{C(\mathbf{H}) > C_{\text{out}}(p)\right\} > p$$

$$C_{\text{ergodic}} = \mathbb{E}\left\{C(\mathbf{H})\right\}$$

Outage Capacity: code over only ***one*** fading channel realization

Ergodic Capacity: code over only ***many*** fading channel realizations

MIMO Capacity Measures

KIM and CHUGG: CAPACITY FOR SUBOPTIMAL RECEIVERS FOR CODED MULTIPLE-INPUT MULTIPLE-OUTPUT SYSTEMS 3311

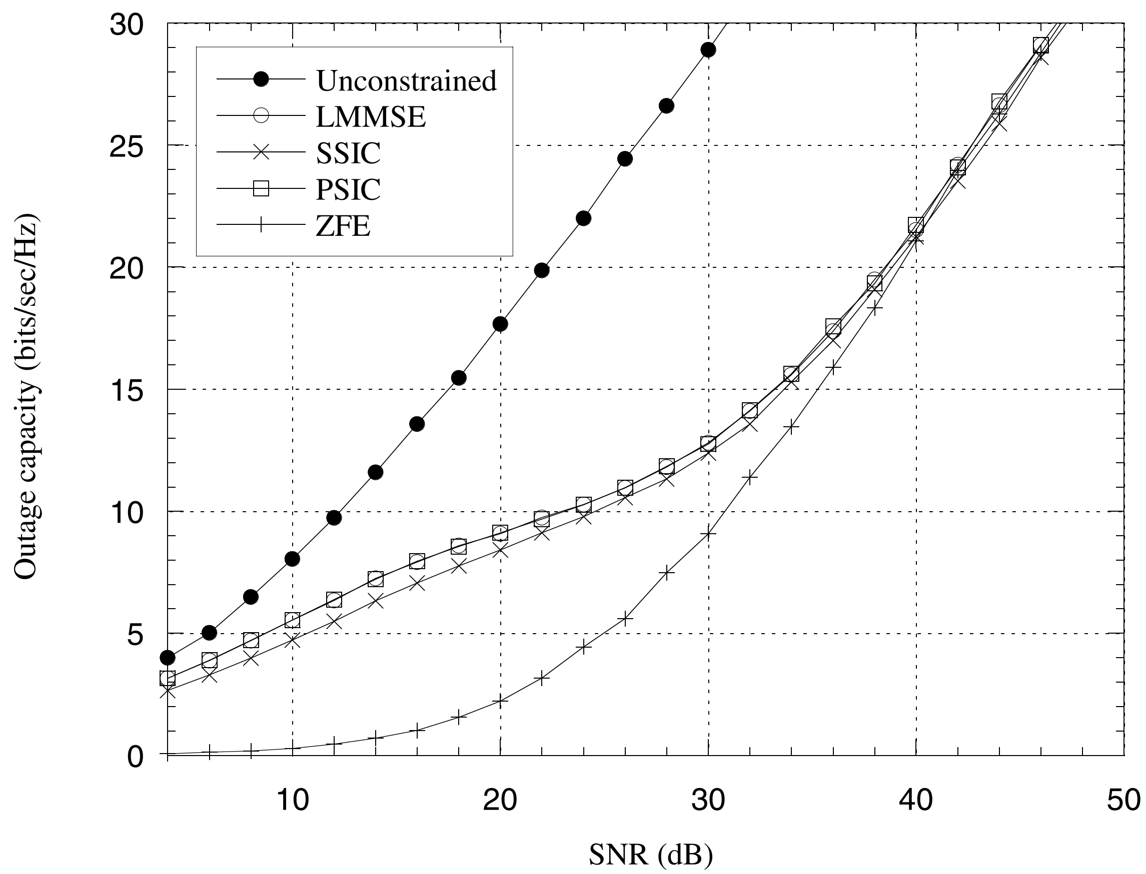


Fig. 5. Outage capacity comparison of decouplers (1% outage probability, N_T=N_R=4).

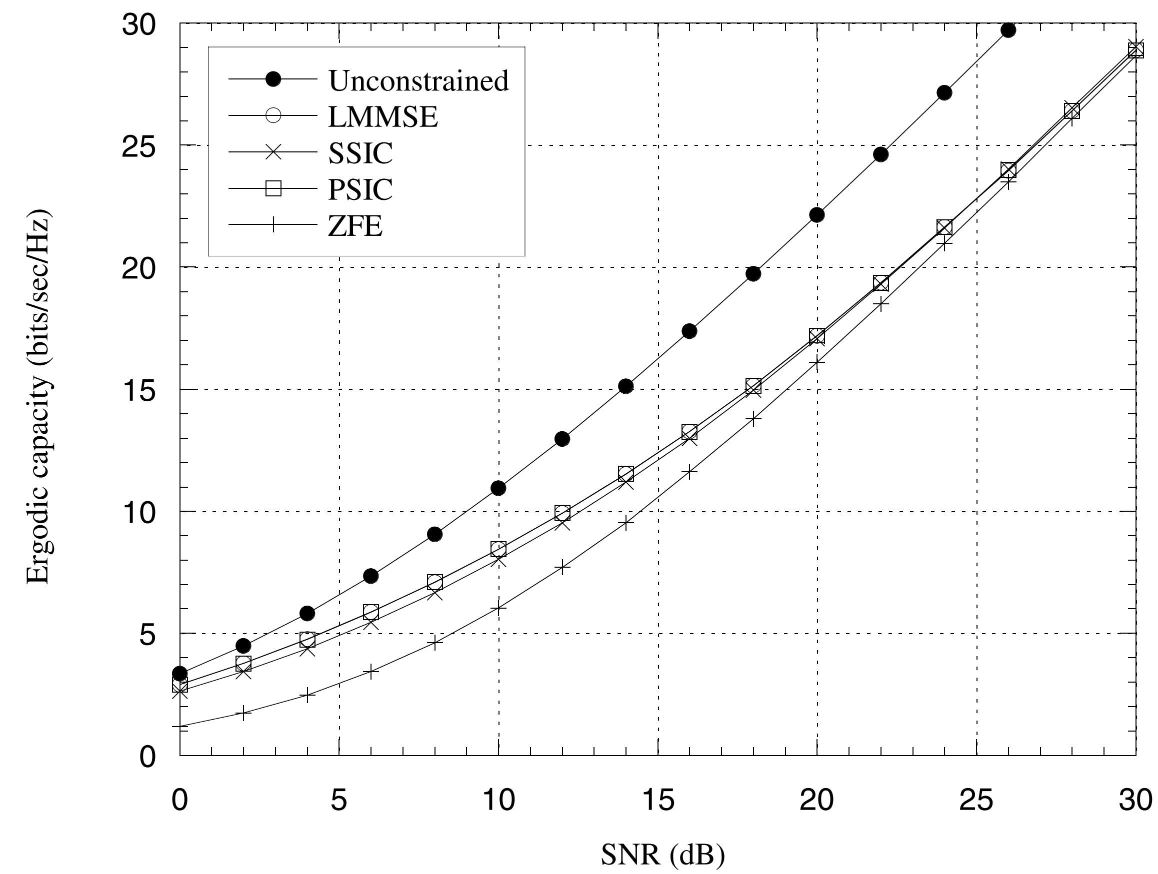


Fig. 6. Ergodic capacity comparison of decouplers (N_T=N_R=4).

Outage Capacity: code over only ***one*** fading channel realization

ST-MUX over Quasi-Static Fading

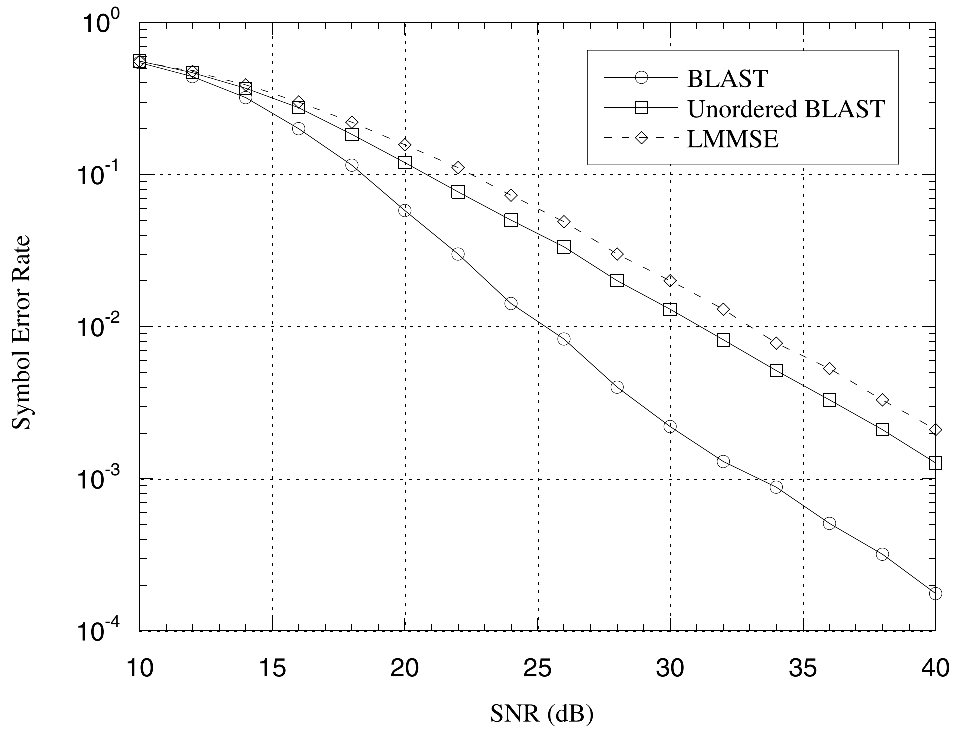


Fig. 7. Uncoded symbol error rates of LMMSE and BLAST (N_T=N_R=4, 16QAM, quasi-static fading).

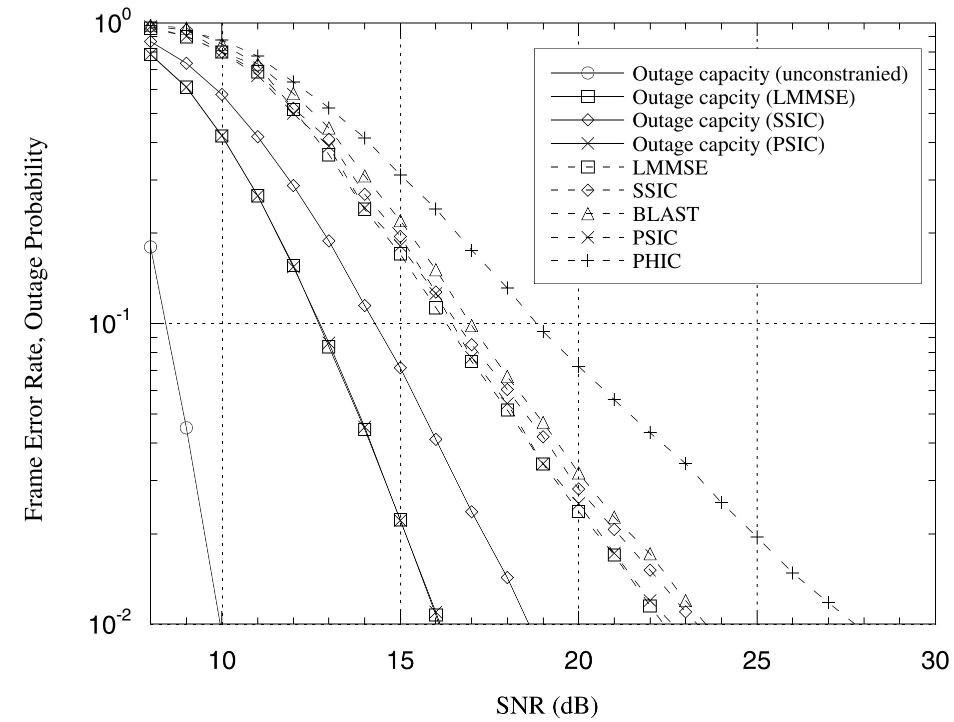


Fig. 9. Performance of Turbo code with LMMSE and BLAST receiver using likelihood. (N_T=N_R=4, 8 bits/sec/Hz, 1022 information bits/frame, quasi-static fading).

uncoded

turbo code

MIMO-OFDM Systems

Modern Code: used over all sub-carriers, over multiple OFDM blocks, over all antennas

Each sub-carrier channel looks like the MIMO channel models that we have considered

Ergodic capacity is a better model when the system gets many orders of diversity — i.e., many coherence BWs, many coherence times, many independent spatial fading modes