Introduction & Motivation

EE564: Digital Communication and Coding Systems

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Course Topic (from Syllabus)

- **Overview of Comm/Coding**
- Signal representation and Random Processes
- Optimal demodulation and decoding
- Uncoded modulations, demod, performance
- Classical FEC
- Modern FEC
- Non-AWGN channels (intersymbol interference)
- Practical consideration (PAPR, synchronization, spectral masks, etc.)

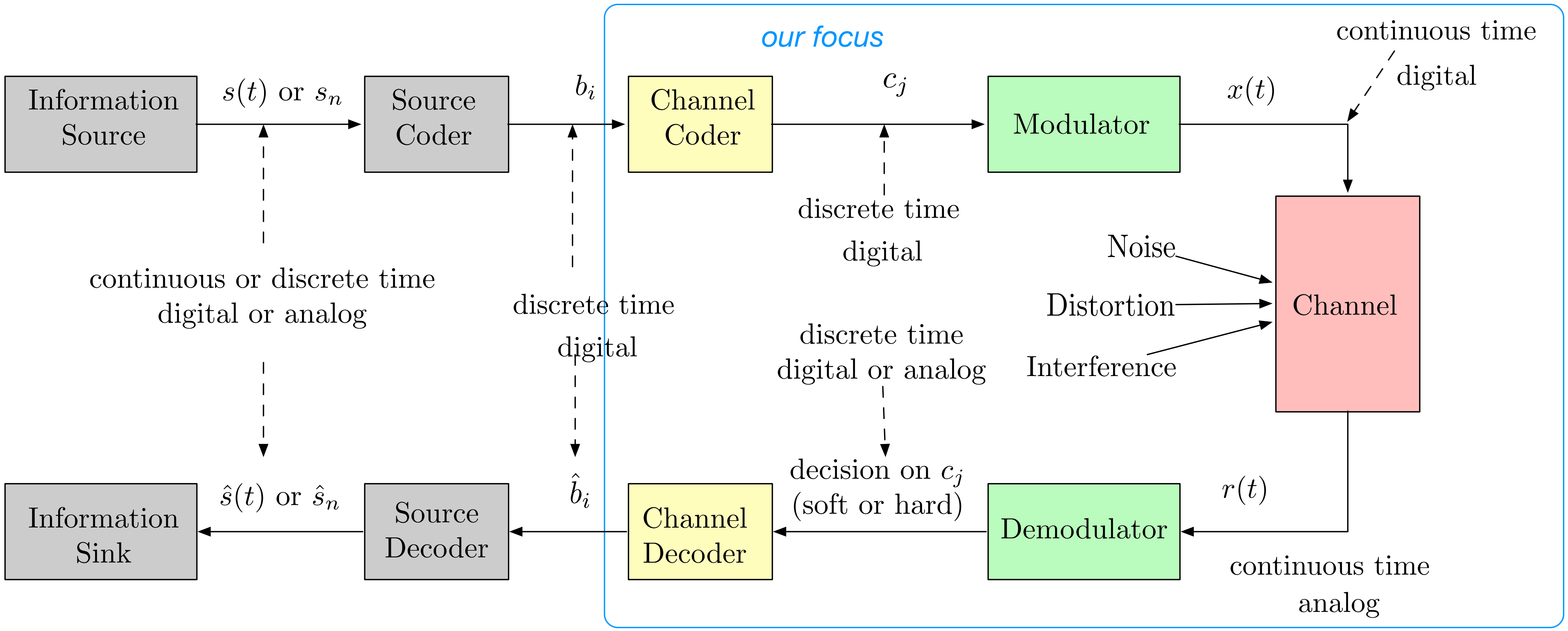
Overview Topics

- Why Digital Comm? Why not analog?
- The digital comm system block diagram
  - Source model and entropy
  - Separation and channel capacity (mutual information)
  - Modulations, Channels, Soft vs. Hard Decision Information
- Performance measures
- Overview of Coding
- More Channels

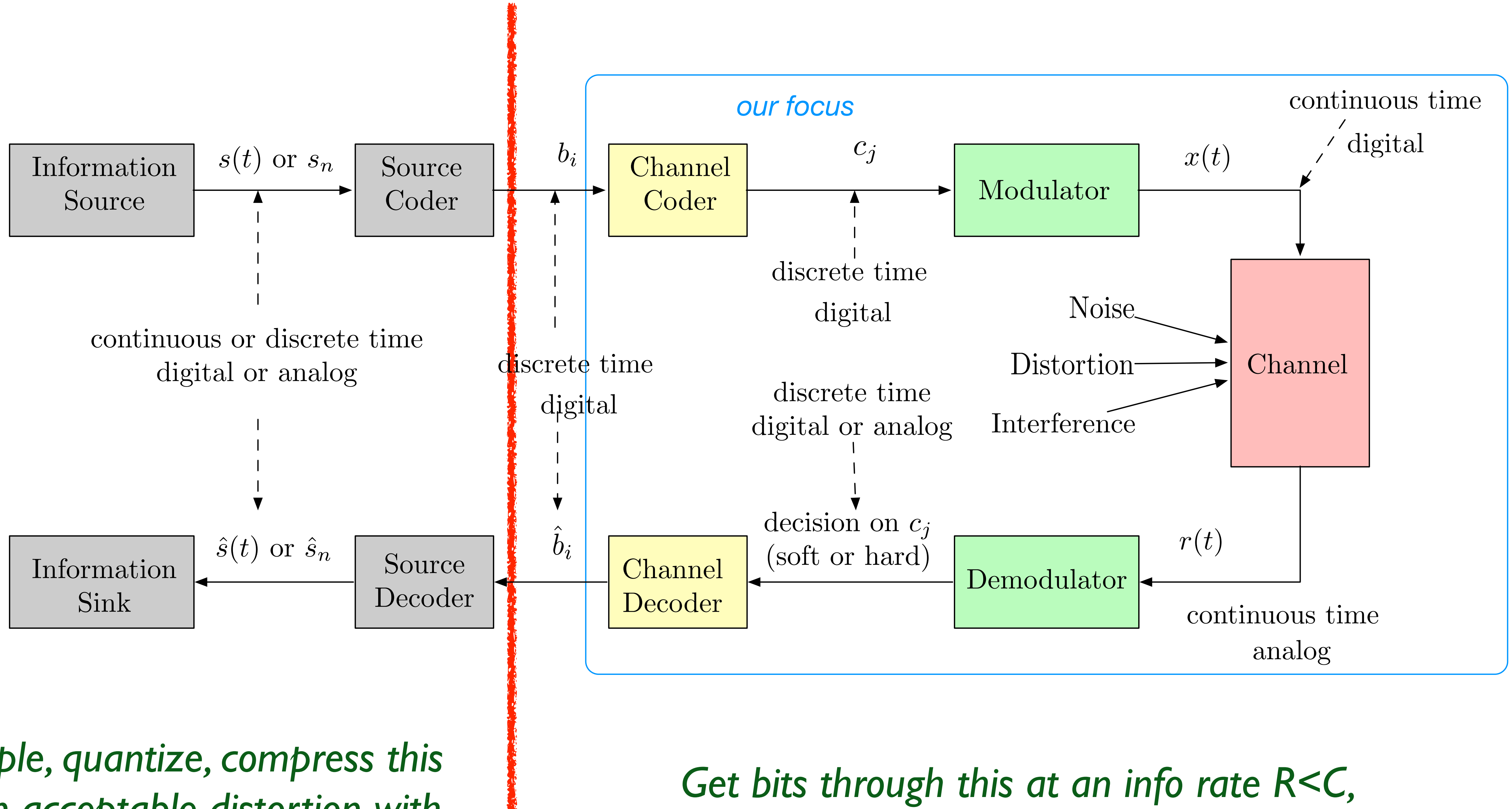
Digital vs. Analog Communications

- Digital comm = send one of a finite number of signals at the transmitter (most modern systems are digital)
- Analog comm = send messages on the continuum (e.g., analog FM radio)
- Why Digital?
  - Exploits digital processing resources (Moore's Law) via ASICs, FPGA, DSP, etc.
  - More robustness and better fidelity — via use of memory in encoding/decoding
  - Control the amount of degradation from source to sink
  - Security (encryption)
  - Easier to share resources: multiplexing, routing, multiple access, multimedia
  - Advantages in multi-hop systems — alleviates distortion accumulating over hops

Digital Comm. Block Diagram



Separation Theorem

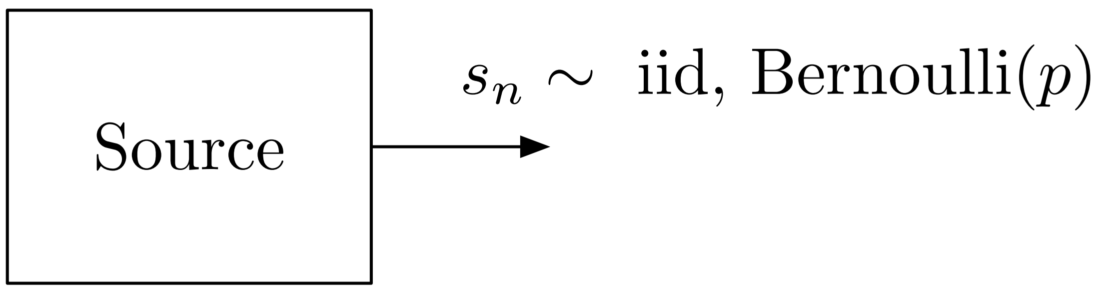


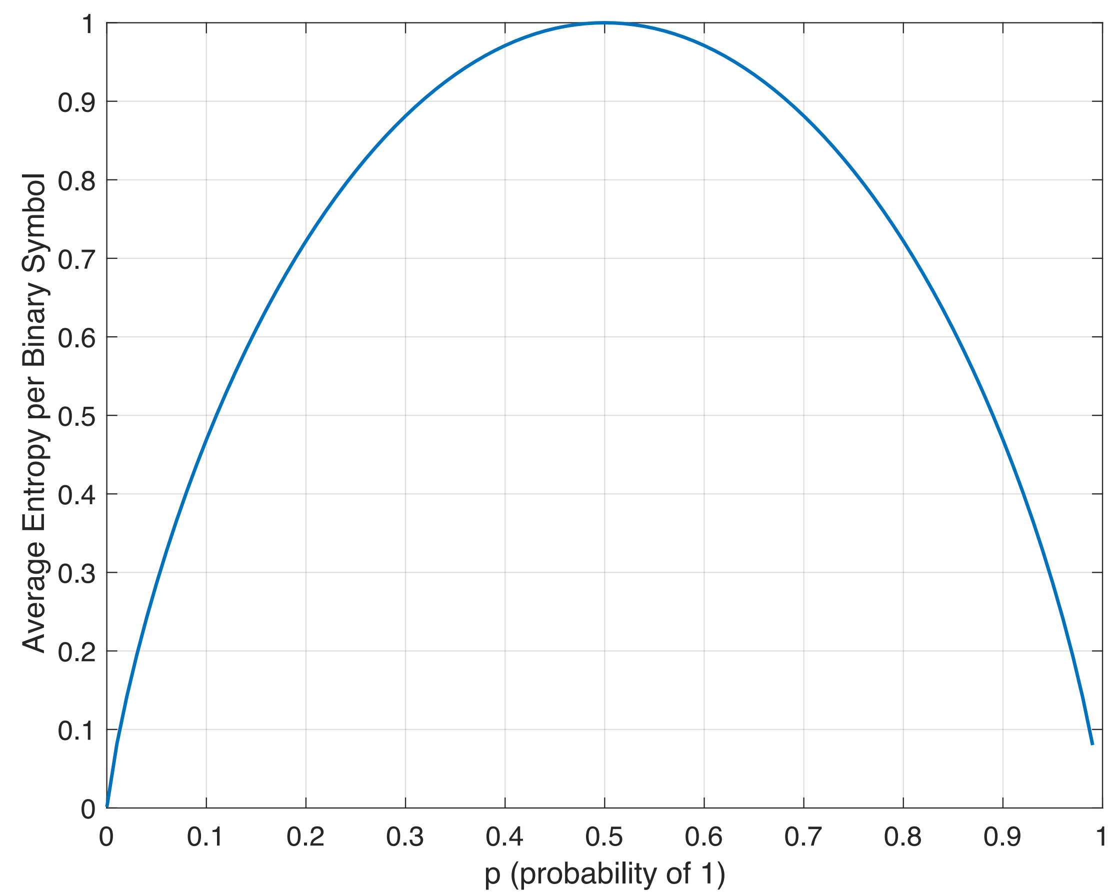
Sample, quantize, compress this to an acceptable distortion with minimal info rate R

Get bits through this at an info rate R<C, C = channel capacity

There is no benefit to combining these tasks if the encoding length for each encoder can be arbitrarily large

Source Model: Binary Memoryless

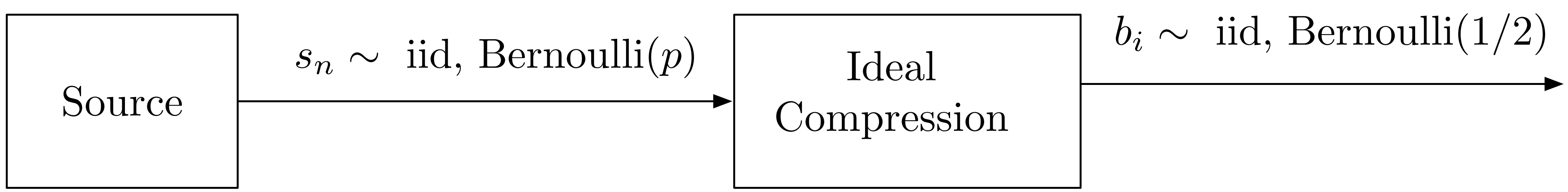




Entropy of the source

$$\mathsf{H}(s_n(u)) = p \log_2 \left(\frac{1}{p} \right) + (1-p) \log_2 \left(\frac{1}{1-p} \right) \qquad \text{(bits/source symbol)}.$$

Lossless Compression

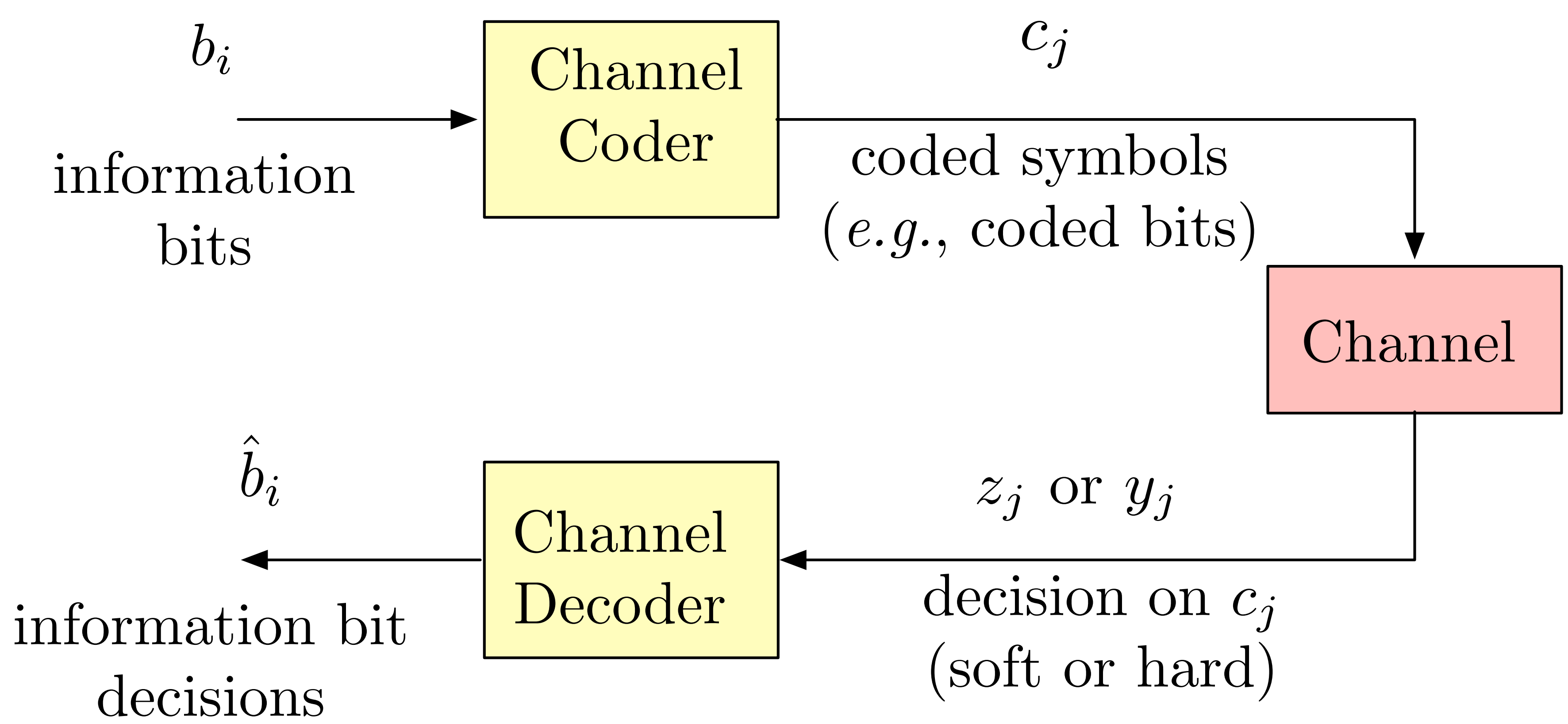


- (Losseless) Source coding theorem:
  - “Source can be compressed to its Entropy and no further”
    - For asymptotically large encoding block size
    - H values of b for each value of s

For EE564, the effective information source is b and it is iid, Bernoulli(0.5)

(we will consider sources with p != 0.5 for iterative decoding)

Coding Block Diagram



This simplified model abstracts the modulation-demodulation and details of the waveform channel

Simplified model is used to study coding

Channel Capacity

Mutual Information

$$
\begin{aligned}
I(x(u); y(u)) &= \sum_y \sum_x p_{x(u),y(u)}(x,y) \left[\log_2 \left(\frac{p_{x(u),y(u)}(x,y)}{p_{x(u)}(x) p_{y(u)}(y)} \right) \right] \\
&= \sum_y \sum_x p_{x(u),y(u)}(x,y) \left[\log_2 \left(\frac{1}{p_{x(u)}(x)} \right) - \log_2 \left(\frac{1}{p_{x(u)|y(u)}(x|y)} \right) \right] \\
&= \underbrace{H(x(u))}_{\text{Entropy in } x(u)} - \underbrace{H(x(u)|y(u))}_{\text{Entropy in } x(u) \text{ given } y(u)}
\end{aligned}
$$

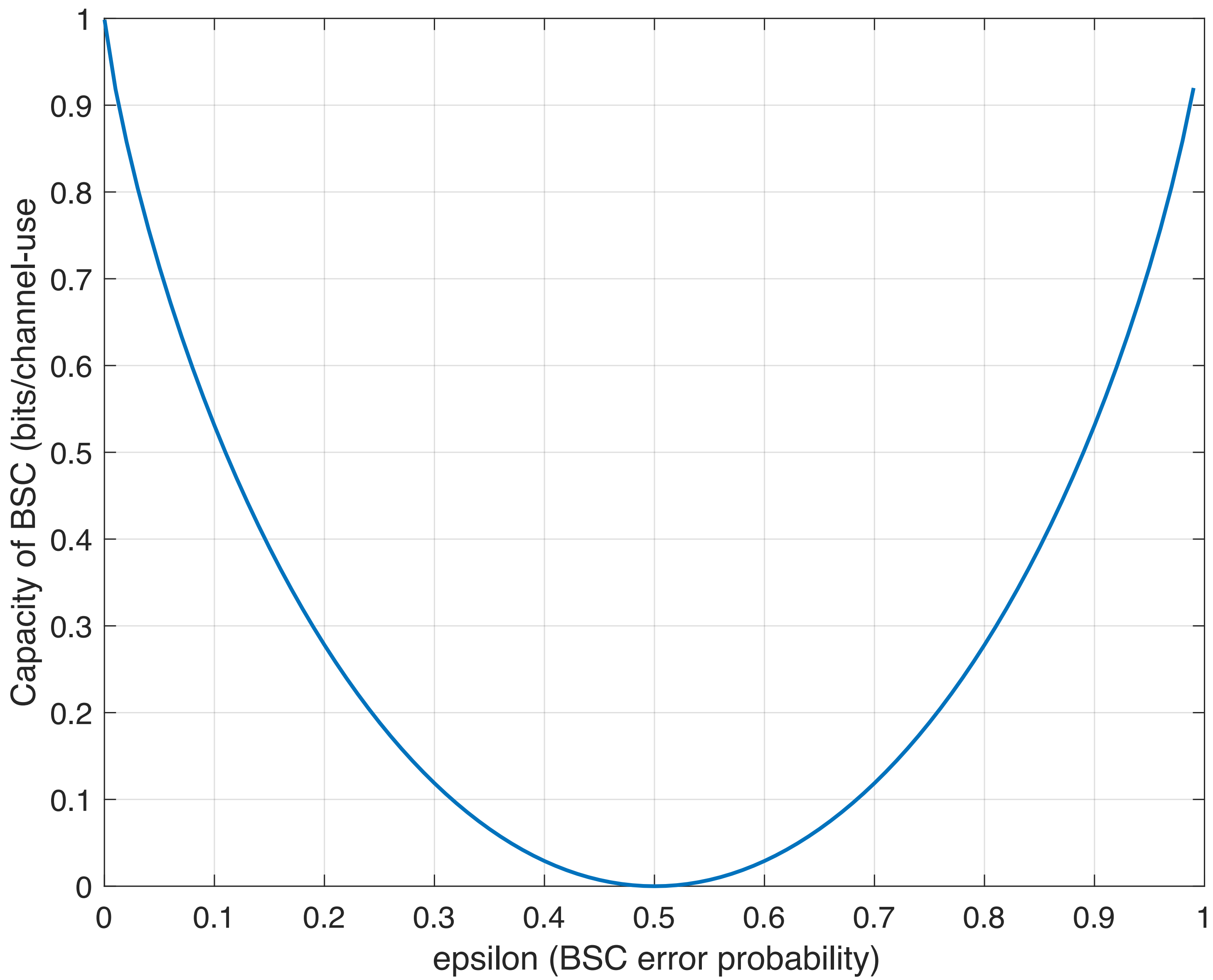
Channel Capacity for Memoryless Channel

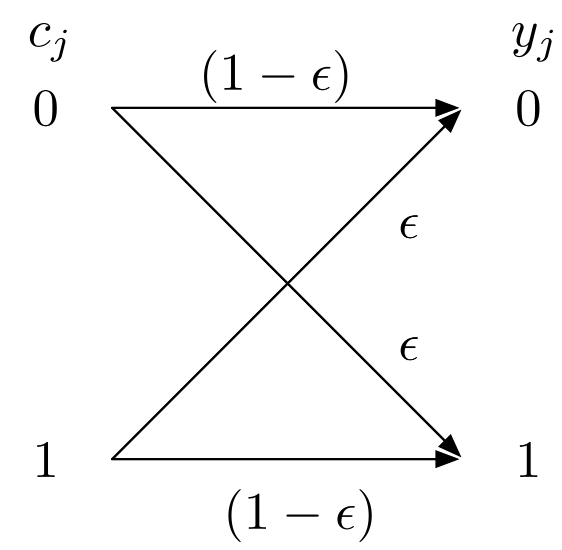
$$
\max_{p_{x(u)}(\cdot)} \mathrm{I}(x(u); y(u))
$$

$$
x_n \longrightarrow \boxed{\text{Channel}} \longrightarrow y_n
$$

$$
P(\mathbf{y}|\mathbf{x}) = \prod_n P(y_n|x_n)
$$

Channel Capacity: Binary Symmetric Channel



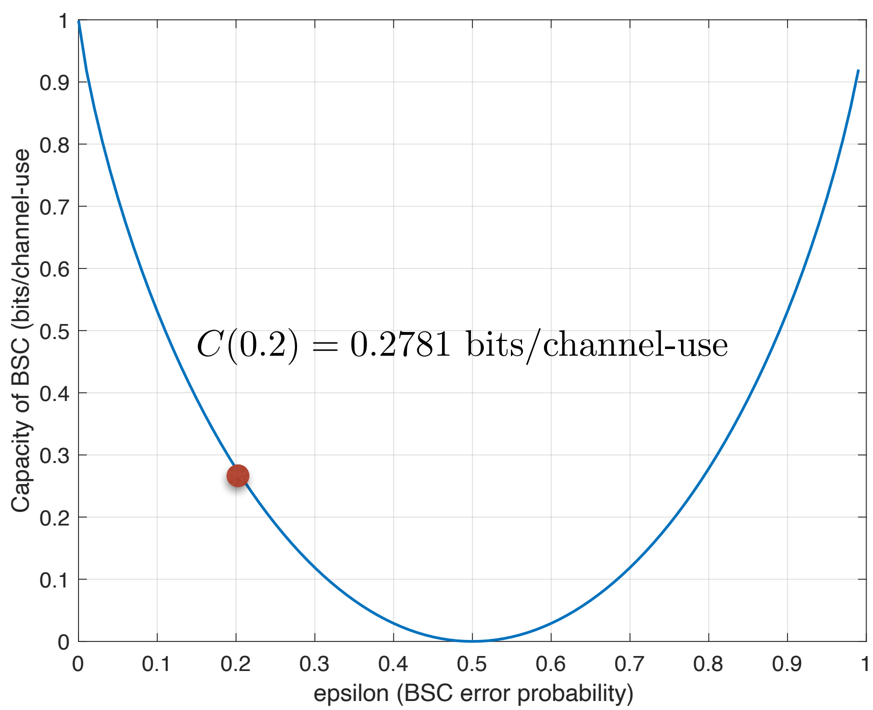


labels: $p_{y_j(u)|c_j(u)}(y_j|c_j)$

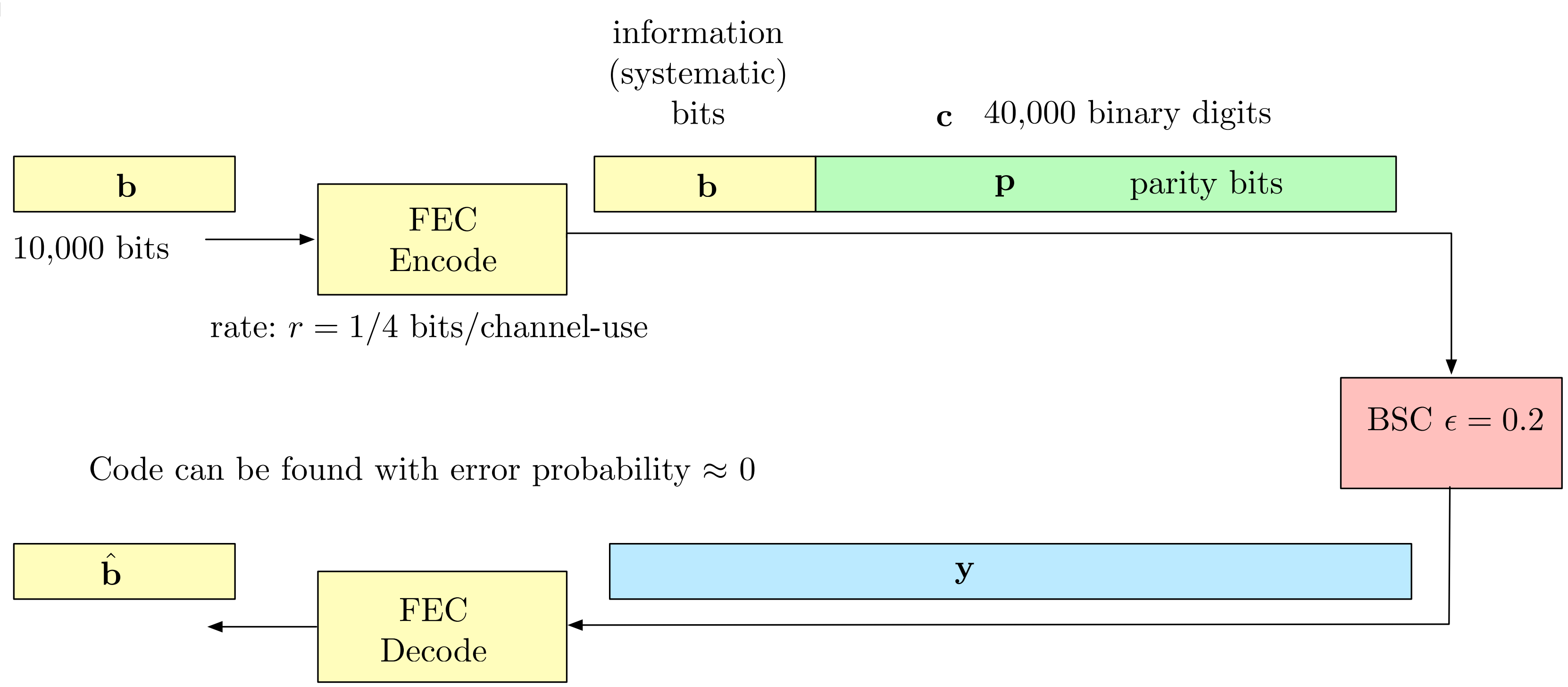
BSC is a special case of discrete memoryless channel (DMC)

$$C(\epsilon) = 1 - H(\epsilon) = 1 + \epsilon \log_2\left(\frac{1}{\epsilon}\right) + (1-\epsilon)\log_2\left(\frac{1}{(1-\epsilon)}\right)$$

Interpreting BSC Capacity

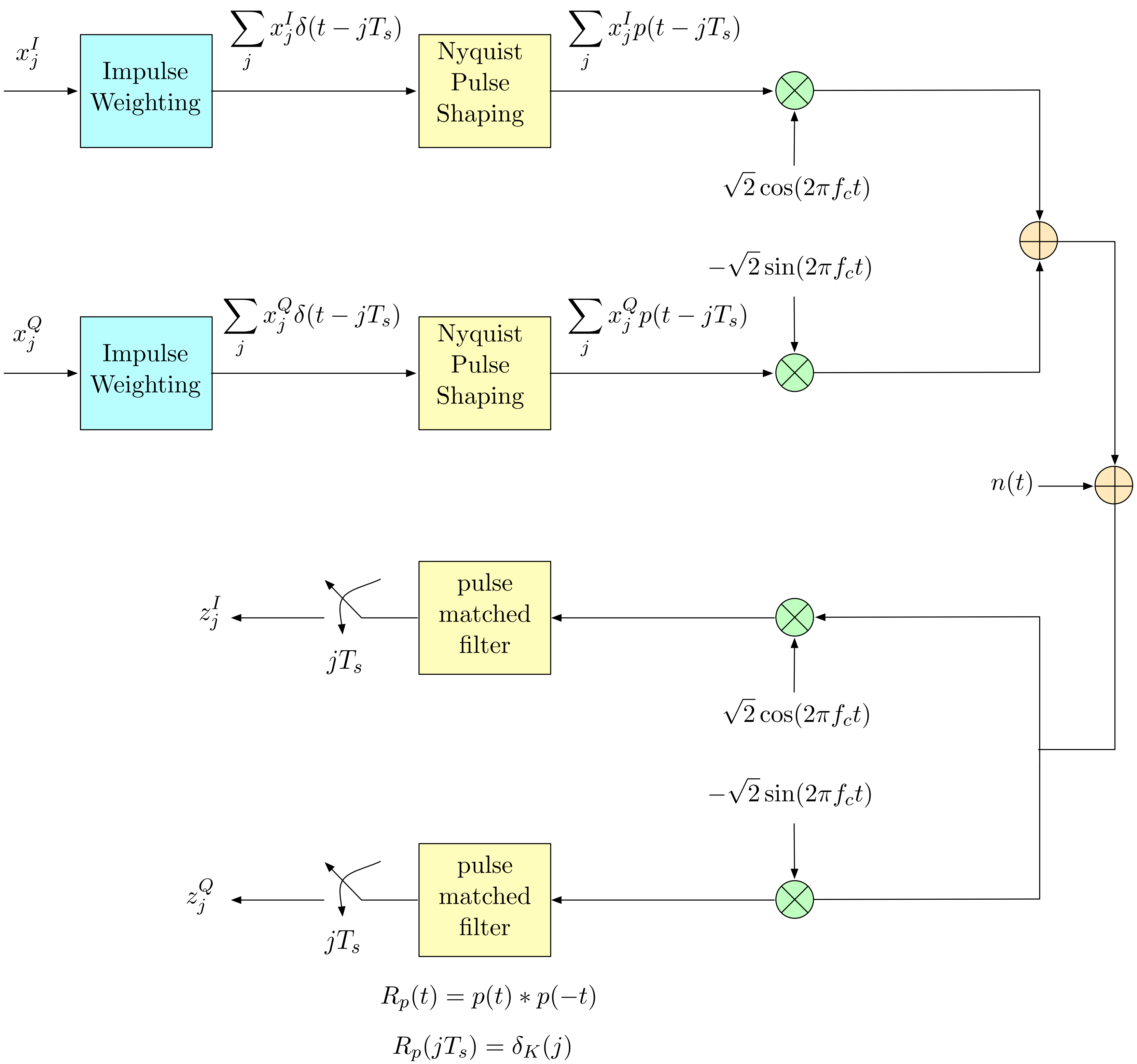


Example of what capacity means



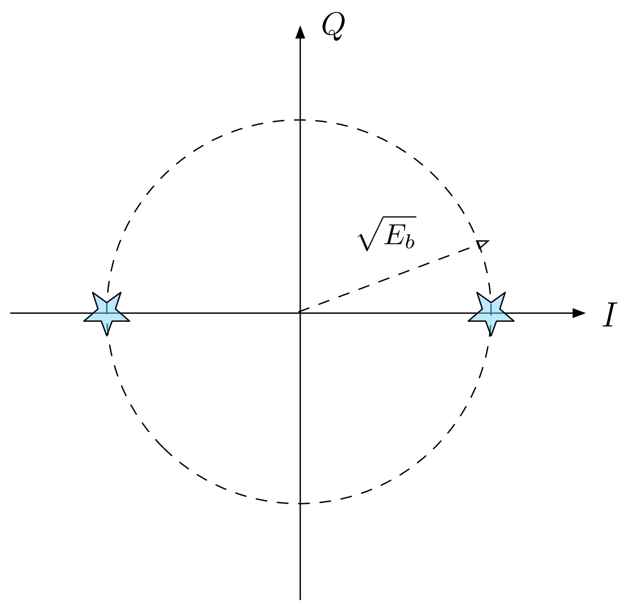
capacity is achieved (approached) through coding

Typical In-Phase/Quadrature Digital Modulation

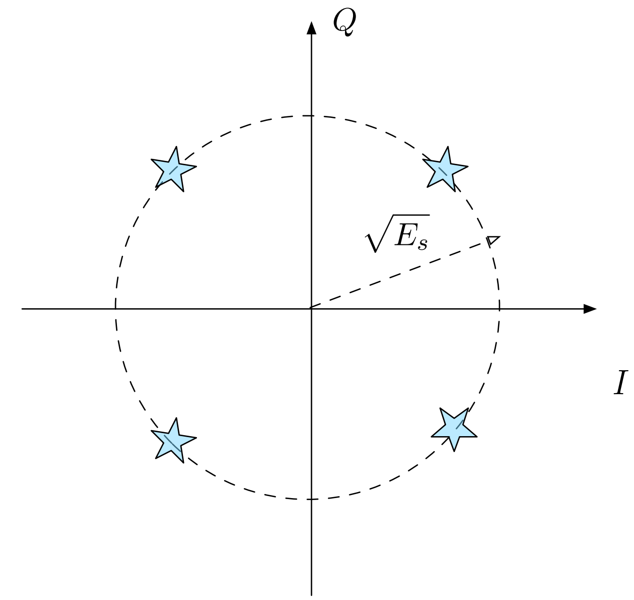


This is approach converts a waveform channel to a 2-dimensional channel vector

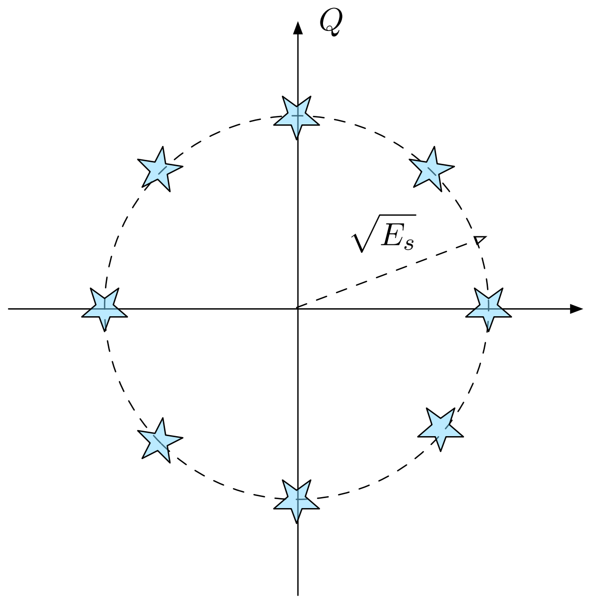
Common I/Q Digital Modulations



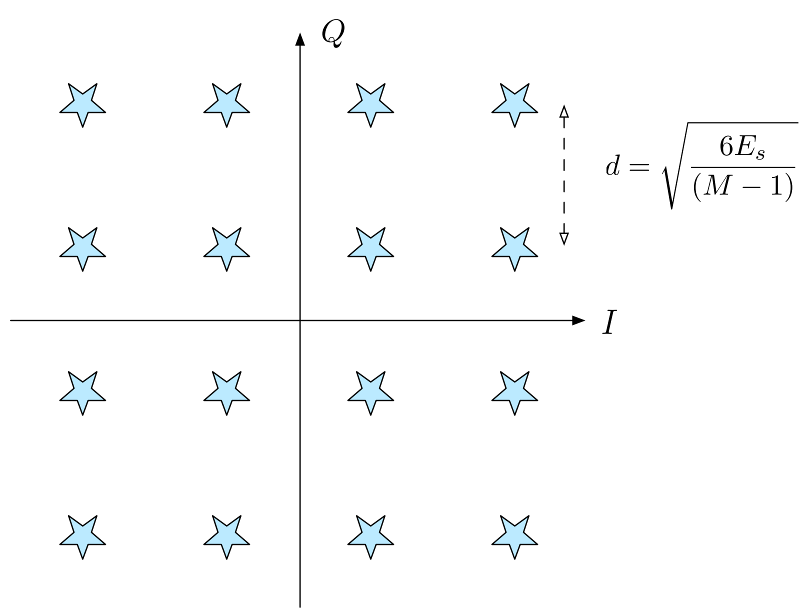
Binary Phase Shift Keying
(BPSK)



Quadrature Phase Shift Keying
(QPSK)



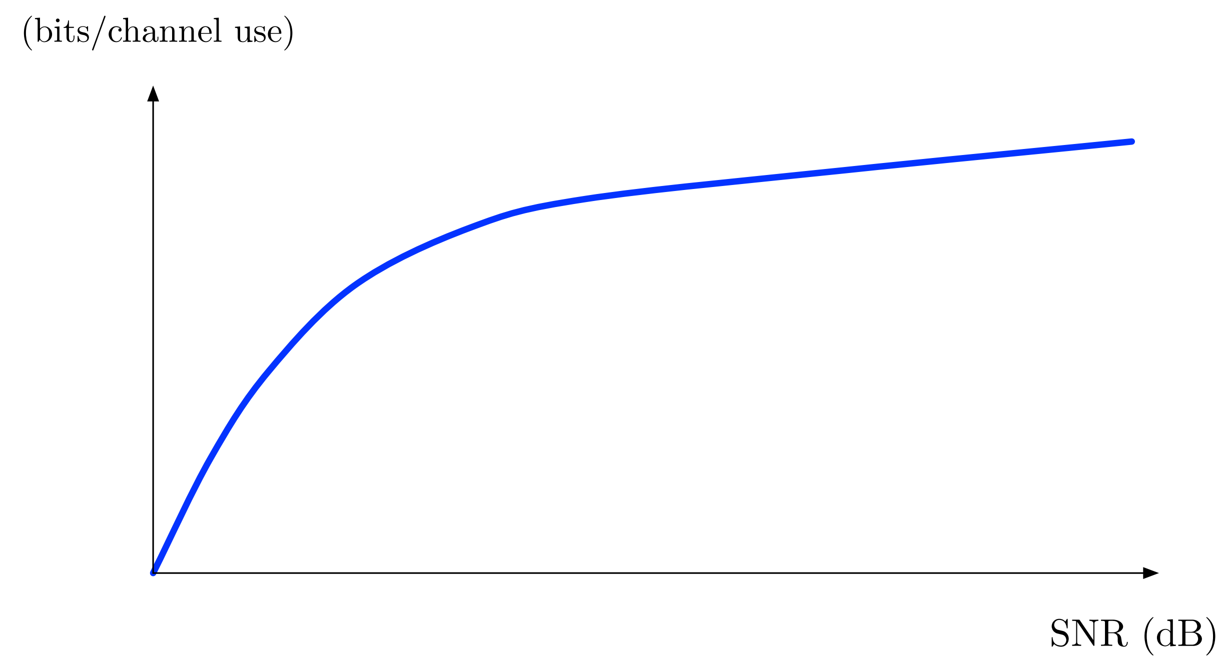
8-ary PSK (8-PSK)

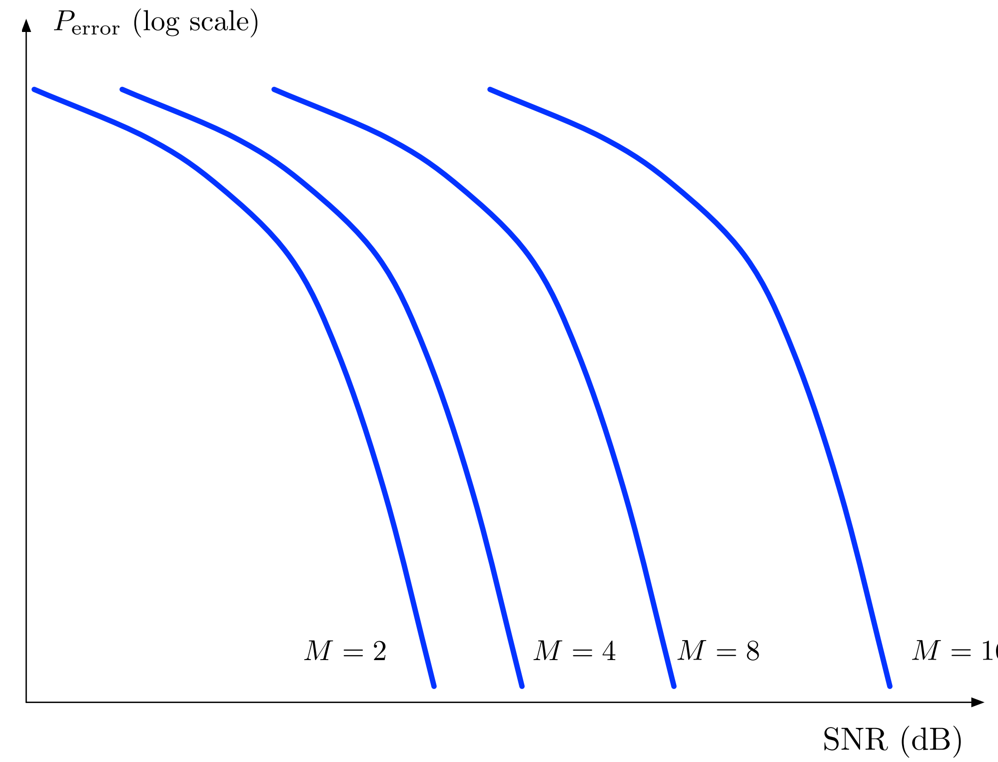


16 Quadrature Amplitude Modulation
(16QAM)

Trade-offs with M?

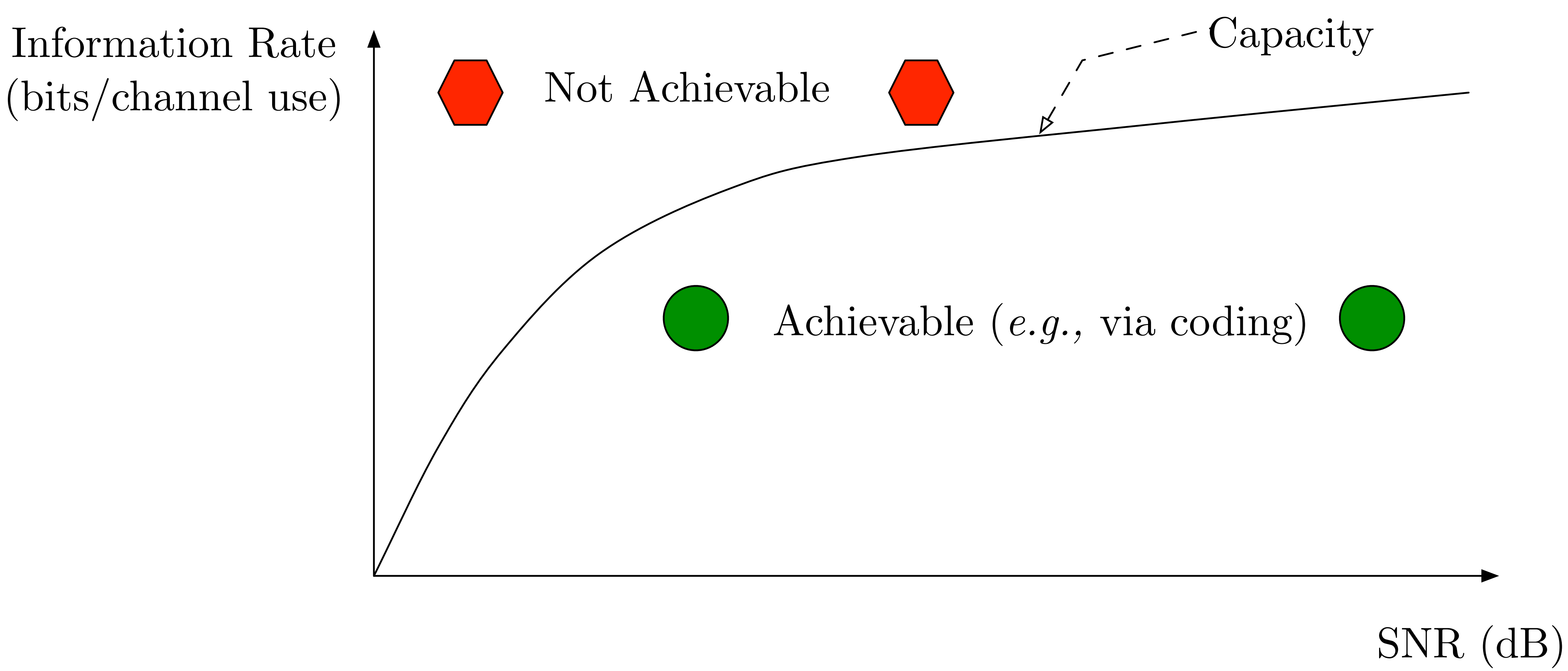
- Dimension = Time * Bandwidth
  - The I/Q constellations use 2 dimensions
- As you increase M
  - More bits/channel use (log2(M))
  - Points get closer for fixed energy





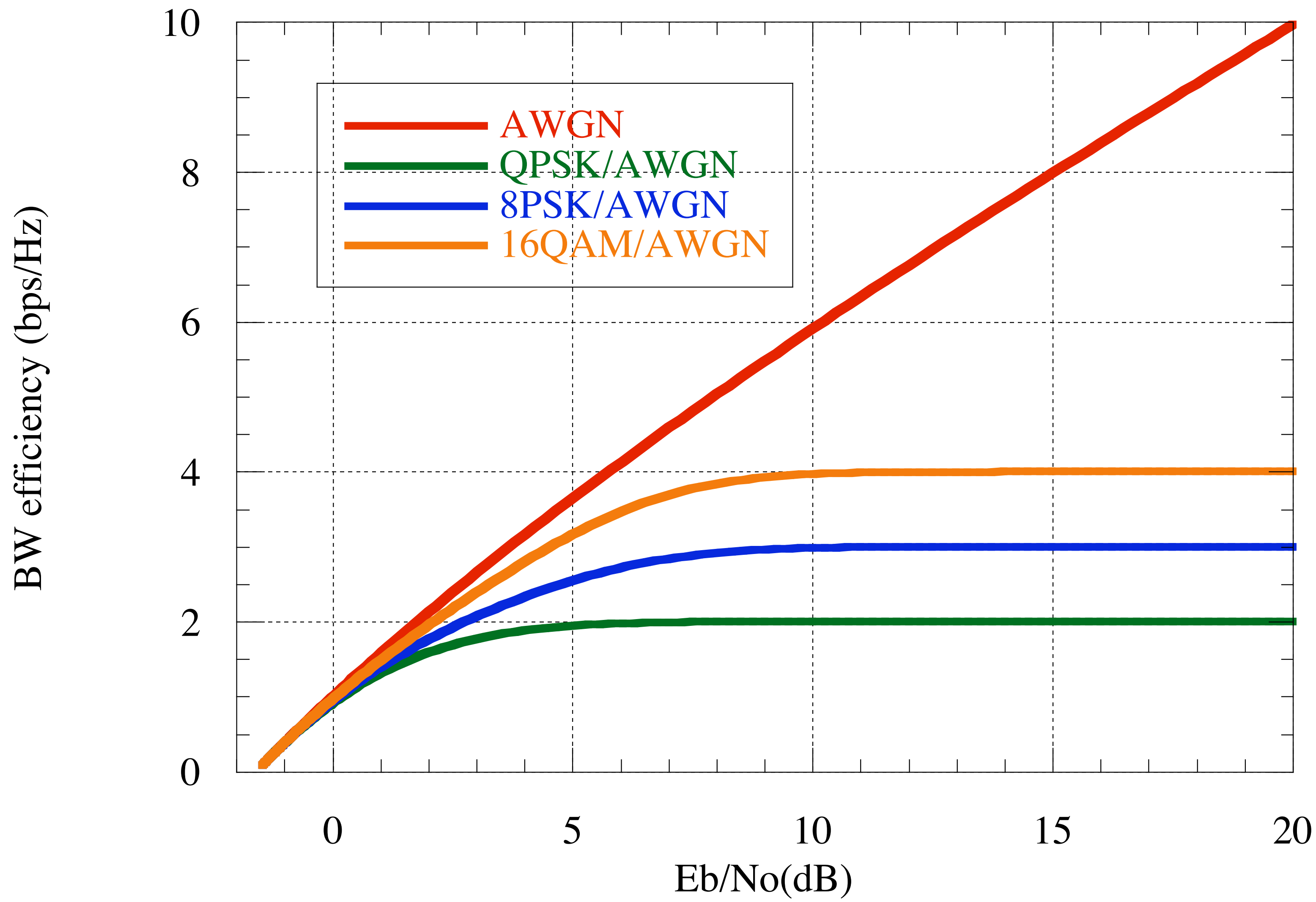
These are the two types of performance plots that we use to evaluate coding and modulation schemes

Throughput vs. SNR Trade (IT)



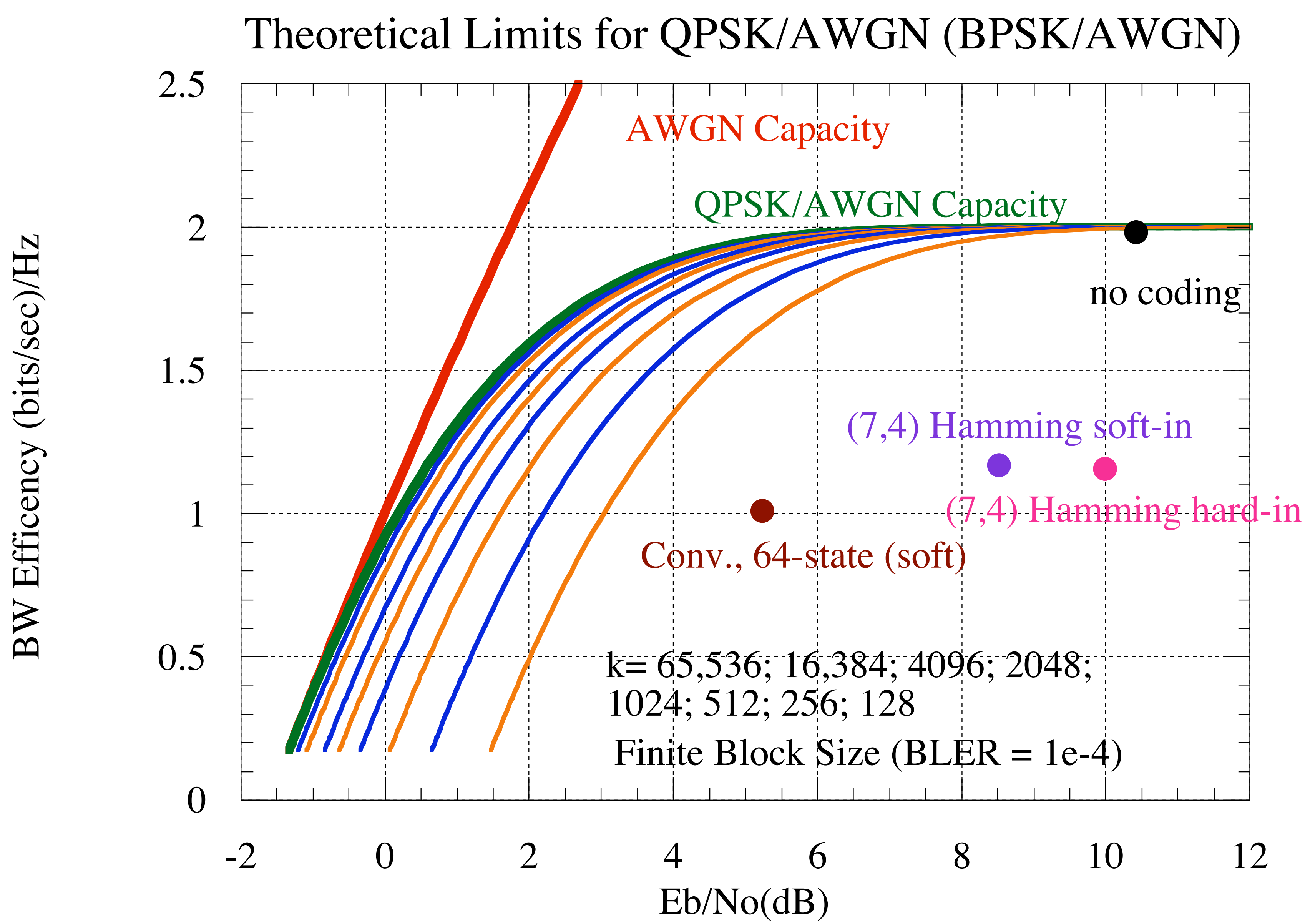
Channel capacity shows up on this type of plot as regions of achievable performance

Throughput vs. SNR Trade (IT)



Example channel capacity curves for AWGN channels with and without modulation constraints

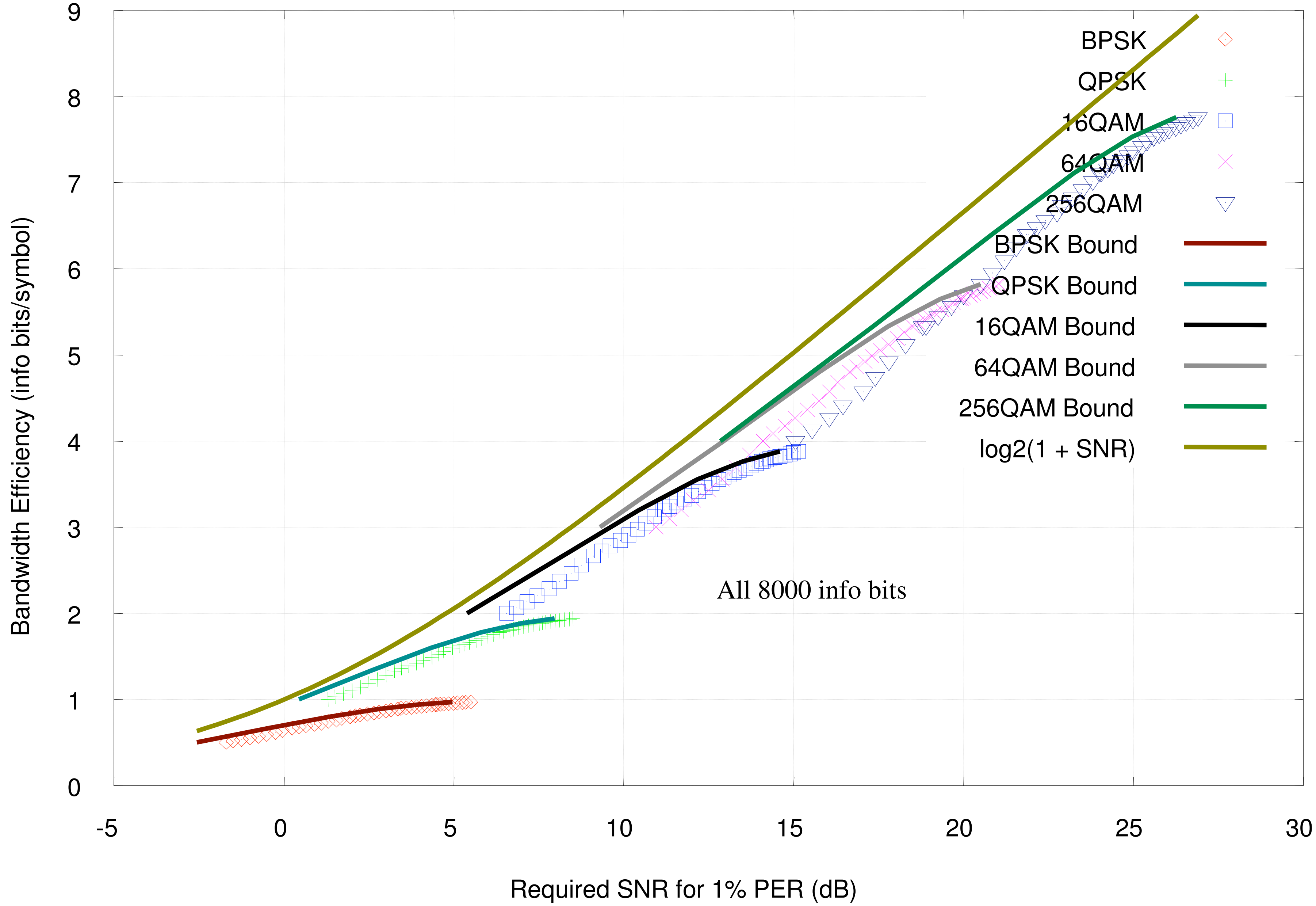
Throughput vs. SNR Trade



How some common coding schemes with BPSK modulation compare to capacity

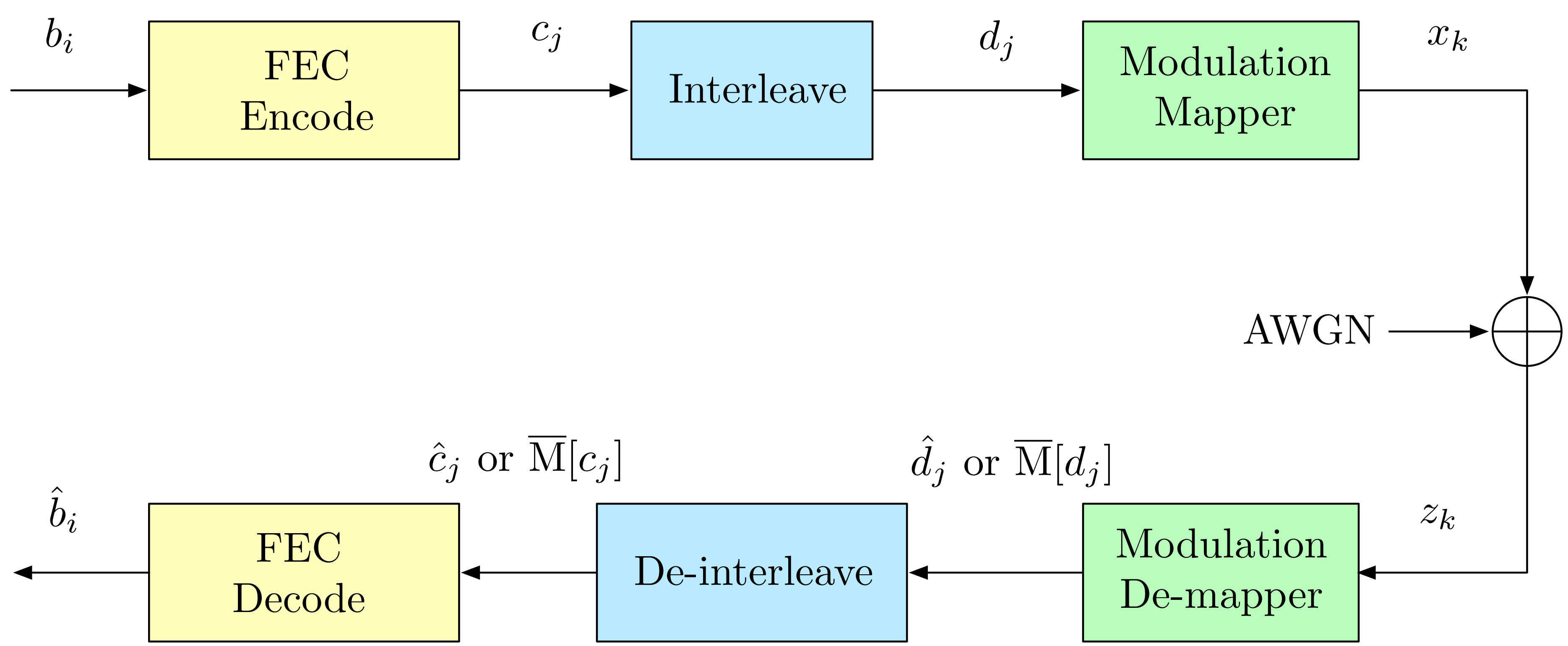
Throughput vs. SNR Trade

AWGN Perf.:Comparison with Bound



An example of a standards contribution based on these same concepts

Bit-Iterleaved Coded Modulation (BICM)



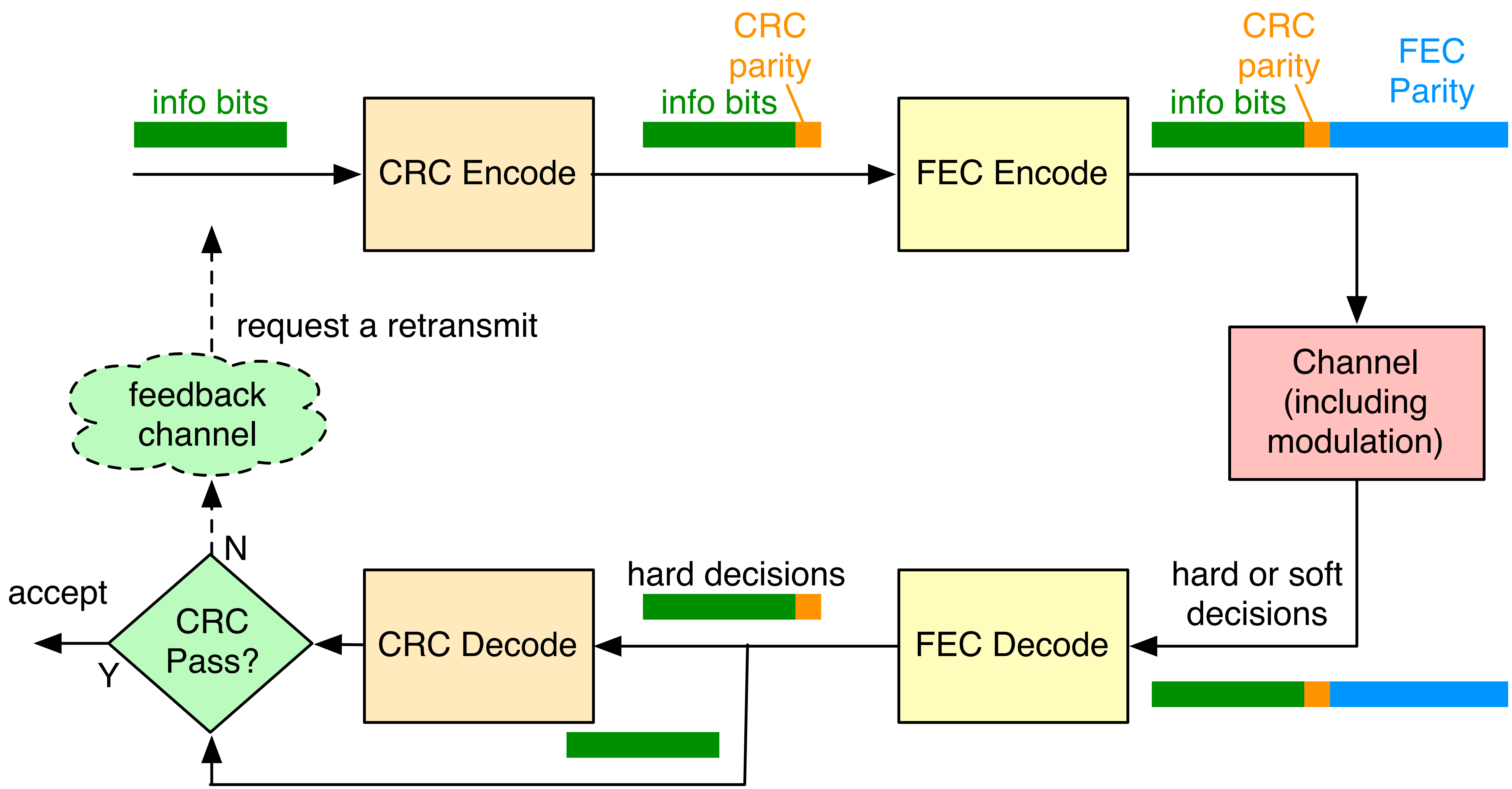
Most common approach to coding & modulation used in practice

(you will simulate a BICM system with a modern code this semester)

Summary of Channel Coding

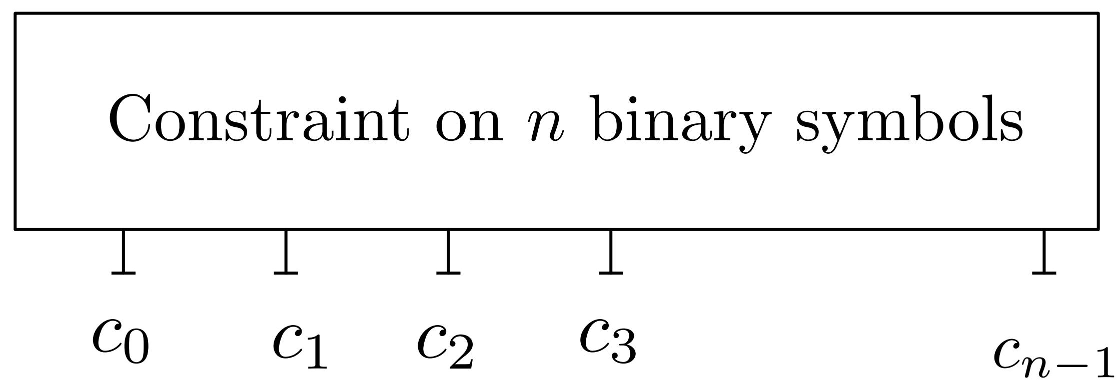
- ***Forward Error Control/Correction Coding (FEC)***
  - As described previously — add redundancy and send across channel
- Error Detection Coding (aka CRC = Cyclic Redundancy Check)
  - Detect if an error has occurred on the channel, but no correction
- Automatic Repeat Request (ARQ)
  - "Hey, I did not get that, send it again!"

Typical Use of Coding in Modern System



Hybrid ARQ (H-ARQ) System

Codes as Constraints on Variables

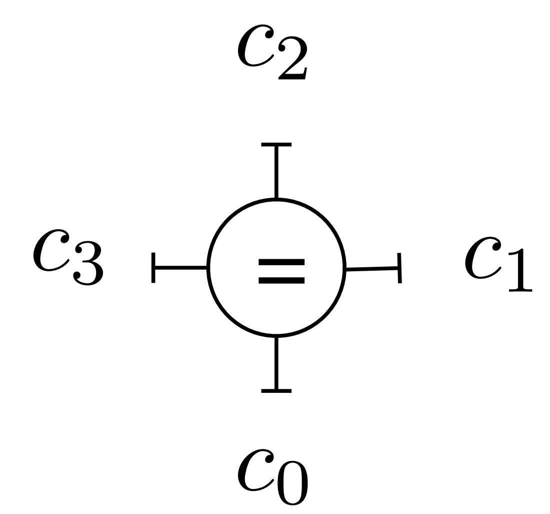


Only 2^k of the 2^n (n x 1) binary vectors are in the code

- ***Repetition Code (equality constraint)***
  - *all n bits are the same*
- ***Single Parity Check Code (SPC code)***
  - *only patterns with even number of 1s*

Example: Repetition Code

Codewords for n = 4: 0000 1111



Number of codewords =2 , so k = 1

rate = 1/n (info bits per channel use)

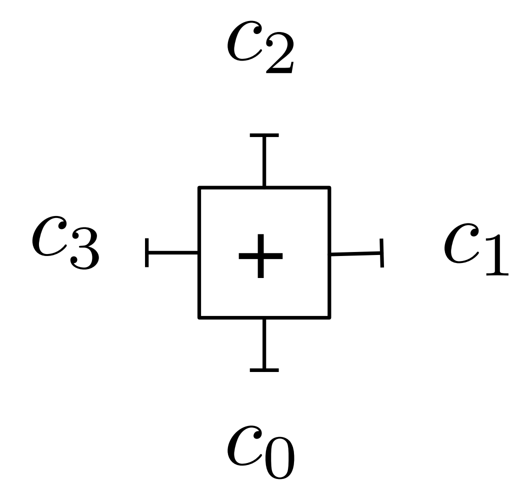
Encoder:

take one information bit in and output n copies of this bit

Example: Single Parity Check Code

Codewords for n = 4:

0000	0101
0011	1001
1100	0110
1010	1111



Number of codewords = 8 , so k = 3 = n-1

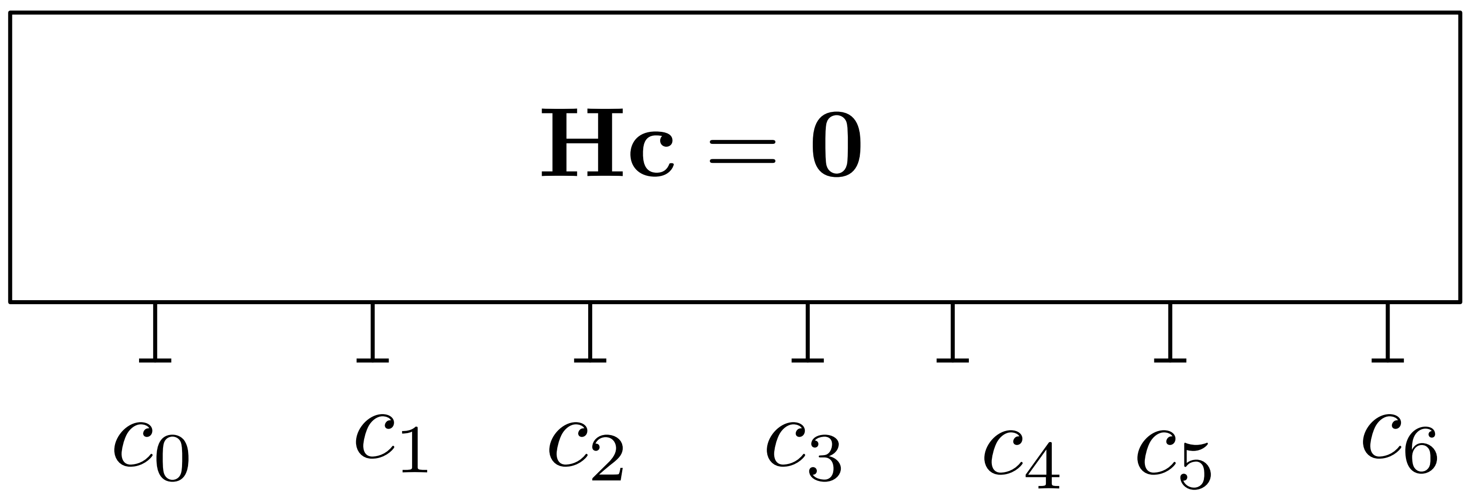
rate = (n-1)/n (info bits per channel use)

Encoder:

take n-1 information bit in and these plus one parity bit which is the mod 2 sum of the input bits

Example: (7,4) Hamming Code

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$



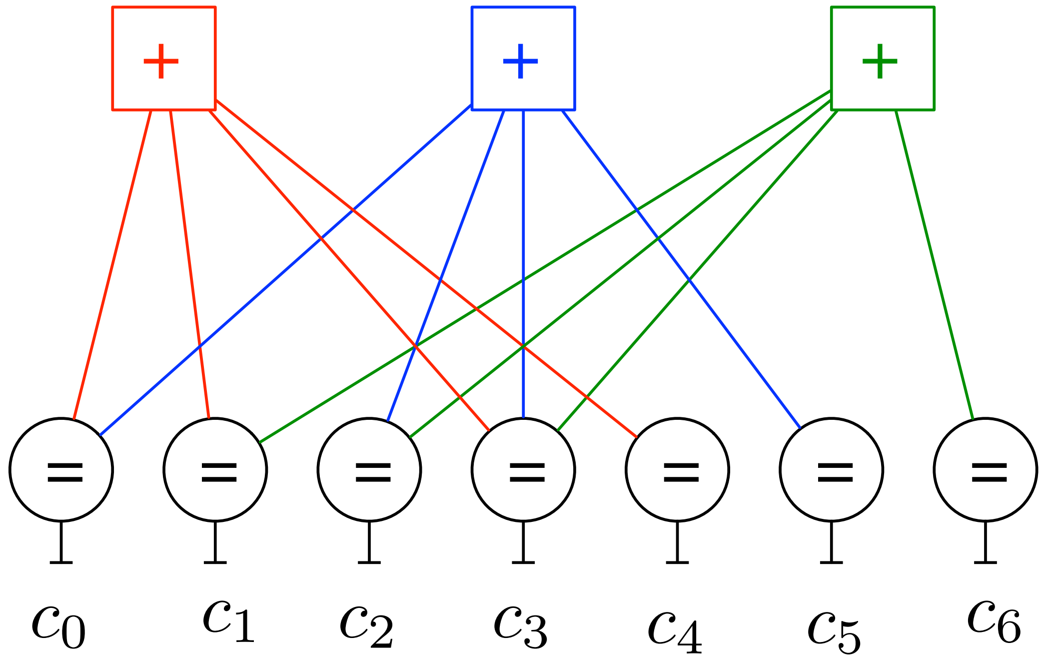
Linear Block Code ("Multiple Parity Check Code")

All three SPCs must be satisfied simultaneously

Example: (7,4) Hamming Code

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Parity Check Graph or Tanner Graph

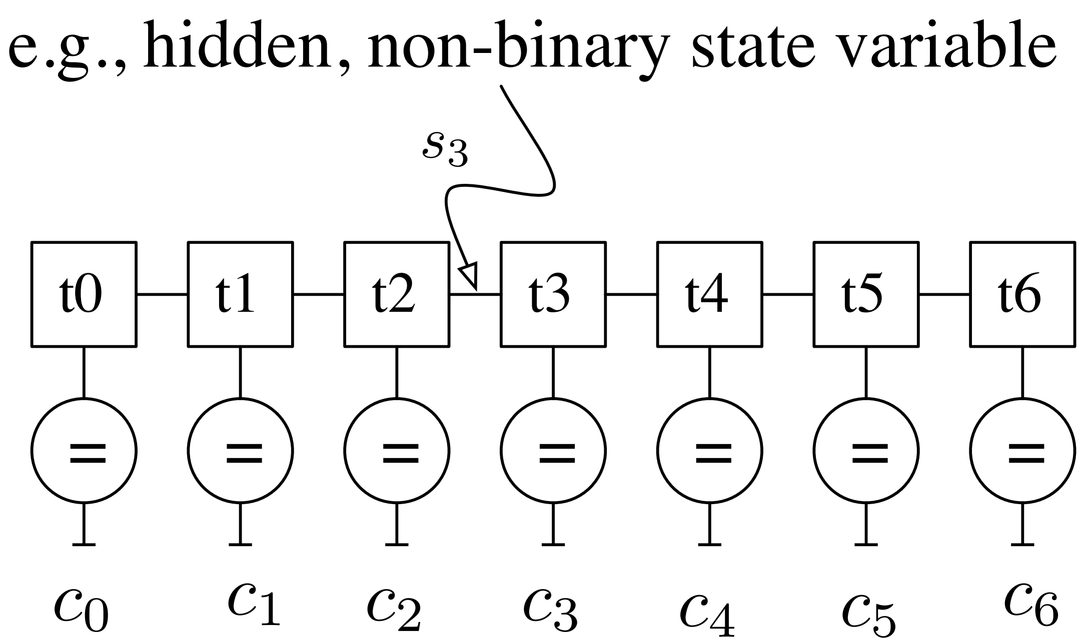


All local constraints must be satisfied simultaneously

Example: (7,4) Hamming Code

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Parity Check Trellis Graphical Model



All local constraints must be satisfied simultaneously

Example: Low Density Parity Check (LDPC) Code

Just a very large (multiple) parity check code with mostly 0s

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & \ldots & 1 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \\ 1 & 1 & \ldots & 0 & 0 & 0 \end{bmatrix}$$

number of 1s = number of bits in first SPC

number of 1s = number of SPCs second code bit is involved in

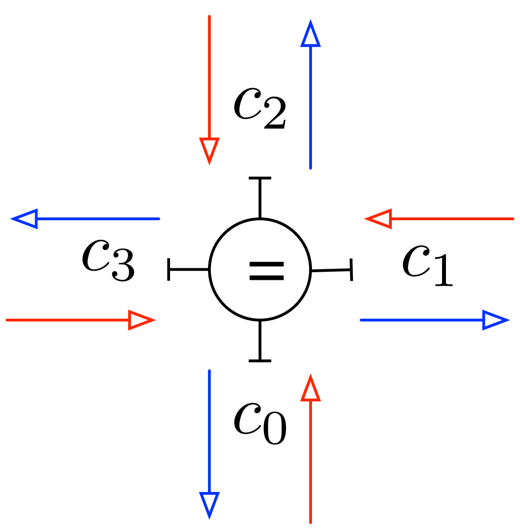
A systematic way to build codes with very large block size

Example: Low Density Parity Check (LDPC) Code

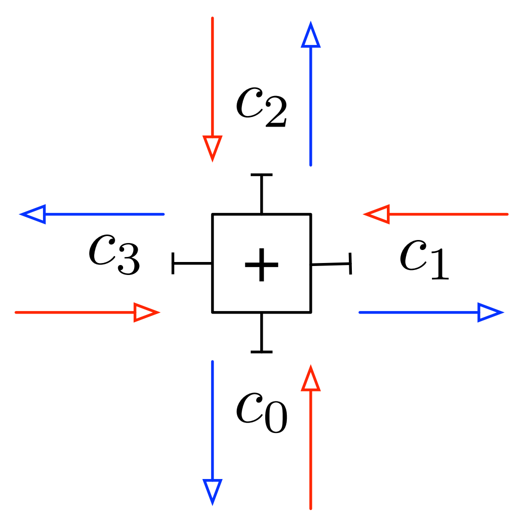
The trick is in the decoding algorithm:

Repeatedly do "soft-in/soft-out (SISO)" decoding of each local code and exchange these soft decisions (messages, beliefs, metrics)

ITERATE until things look good!



Equality Constraint SISO



SPC SISO

SISO rule
(message update rule)
depends on code constraint

Incoming soft-decision information

Outgoing soft-decision information

Example: Low Density Parity Check (LDPC) Code

This simple construction with this decoding approach can approach channel capacity with large block sizes

Summary of Modern FEC

- **Construct large codes (big n) by connecting simple, local (or constituent) codes via pseudo-random permutations**
- **Iteratively decoding**
  - ***Run SISO decoding for each local code***
  - ***Exchange soft-information between local code SISOs***

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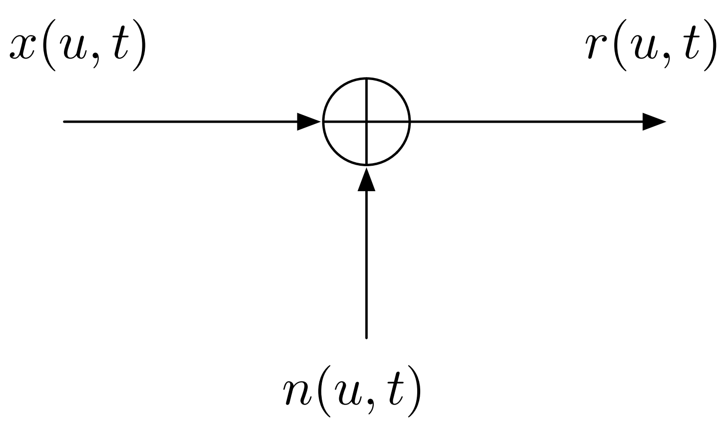
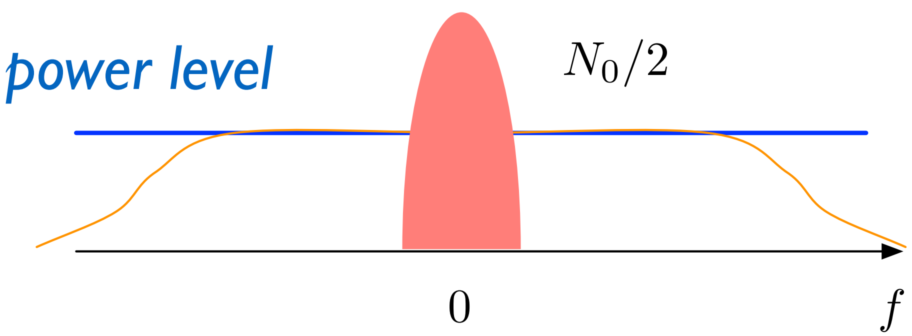
- Performance measures
- Overview of Coding
- More Channels

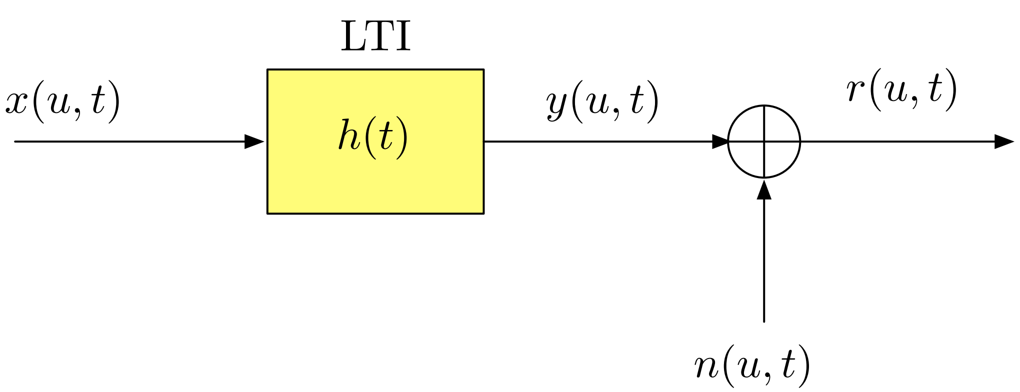
More on Channel Models

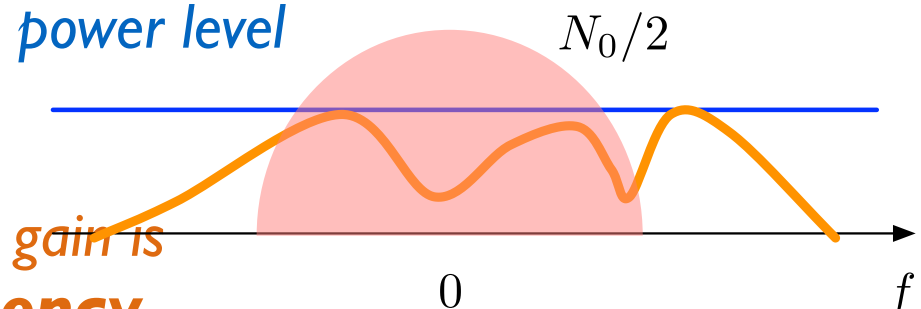
AWGN





AWGN - Intersymbol Interference (ISI) Channel





Channel Models

- We will focus primarily on the AWGN channel
- Several approaches to ISI-AWGN
  - Convert to many parallel, narrow, frequency channels with each having flat gain: ***Orthogonal Frequency Division Multiplexing (OFDM)***
    - *Converts to many parallel AWGN channels*
  - Use a constrained receiver structure such as a linear filter to try to invert ISI effects: ***(Linear) Channel Equalization***
  - Do optimal data detection with ISI channel modeled: ***MAP Sequence/Symbol Detection — Viterbi for hard-out and Forward-Backward Algorithm for soft-out***
    - *We will learn these algorithms as part of the coding material*