EE535 Homework Problems – Keith M. Chugg

September 23, 2003

1. Comparison shopping. Go to a mobile phone store and get answers for the following questions:
   - "Digital" vs. Analog, which one is better?
   - Is their phone
     - CDMA phone?
     - TDMA phone?
       - North American Standard?
       - European Standard?
   - PCS vs. Cellular?
     - PCS ⇒ what is the coverage?
   - Cost?
   - What is a dual-mode phone? What's a dual-band phone?
   - What wireless web services are available? What data rate?
   - What is a 3G system and when will it be available?

2. The goal of this problem is to help establish the effects of CCI. Assuming a particular binary modulation scheme and a single, synchronous co-channel interferer, the effect of CCI can be model by

$$Z(u) = B(u)\sqrt{E_b} + C(u)\sqrt{E_c} + W(u)$$

   where $B(u)$ and $C(u)$ represent a single bit of the user-of-interest's and the interferer's data, respectively. Specifically, $B(u)$ and $C(u)$ are independent and each take on $+1$ and -1 with probability $1/2$. The effects of the channel noise are summarized through the additive Gaussian noise $W(u)$, which has zero mean and variance $N_0/2$. The terms E_b and E_c model the received energy per bit for the user and interferer, respectively.

   (a) The receiver estimates $B(u)$ by $\hat{B}(u)$ by deciding that $\hat{B}(u) = 1$ if $Z(u) > 0$, otherwise it decides that $\hat{B}(u) = -1$. Find the probability of error (i.e., $\Pr\left\{\hat{B}(u) \neq B(u)\right\}$) as a function of the SIR ($\alpha = E_b/E_c$) and E_b/N_0.

   (b) Plot the probability of error vs. E_b/N_0 for various values of the SIR (i.e., SIRs of ∞, 40 dB, 30 dB, 20 dB, 10 dB, 5 dB, 0 dB). Use a log scale for the probability of error and plot E_b/N_0 in dB (i.e., $10\log_{10}(E_b/N_0)$).

   (c) Discuss the limiting performance as $N_0 \to 0$.

(d) For moderate to high E_b/N_0, the performance degradation due to CCI can be approximated as a loss in effective E_b/N_0. Find an expression for this loss and plot it versus α in dB.

3. Methods for combating CCI are of interest because they increase the capacity in TDMA (or FDMA) cellular systems by allowing for smaller reuse factor N_c. Specifically, with all other aspects of the system design held fixed, changing the reuse factor from $N_{c,old}$ to $N_{c,new}$ increases the capacity roughly by a factor of $N_{c,old}/N_{c,new}$. Assume a baseline system with $N_c = 7$.

   (a) What is the increase in the worst case C/I in dB associated with $N_c = 4$ and $N_c = 3$ when $\beta = 3$?

   (b) Repeat (3a) for $\beta = 4$.

   (c) What is the increase in capacity for $N_c = 4$ and $N_c = 3$, respectively?

   (d) Generalize the results of (3a)–(3c) by plotting capacity gain vs. χ, where χ is the decrease in the minimum required value of C/I in dB for a given CCI-rejection algorithm. Produce plots for both $\beta = 3$ and $\beta = 4$.

4. Repeat (3d) for 120 degree sectoring.

5. Consider the case when there are K interferers yielding an SIR of

$$\text{SIR} = \frac{P(u)}{\sum_{k=1}^{K} P_k(u)}$$

   Consider the case where each interferer has equal power according to the path loss model – i.e., SIR_{PL}.

   (a) For $\text{SIR}_{PL} = 20$ dB, compute the value of $\bar{P}_{dB}$ and $\bar{P}_{dB}(k)$ for $K = 1, 2, 3, 4, 5, 6$.

   (b) When shadowing is considered, the value of the SIR is a random variable. Again, for $\text{SIR}_{PL} = 20$ dB, plot a histogram of of SIR for $K = 1, 3, 6$ (i.e., generate realizations on a computer). Consider a log-normal shadowing model with $\sigma_{dB} = 8, 4$. Each interferer and the signal-of-interest are shadowed independently. This means that you'll produce 6 different histograms. In addition to your histograms, **briefly** summarize how you generated the data.

   (c) Does an increase in the shadowing deviation mean that the SIR value varies more?

   (d) How does the variation in SIR depend on the number of interferers?

6. Consider the log-normal random variable $P(u)$ given by

$$P(u) = 10^{(\bar{P}_{dB} + \epsilon_{dB}(u))/10}$$

   where $\bar{P}_{dB}$ is a constant and $\epsilon_{dB}(u)$ is Gaussian with zero mean and variance $\sigma^2_{\epsilon_{dB}}$.

(a) Find $\mathbb{E}\{[P(u)]_{dB}\}$

(b) Find $[\mathbb{E}\{P(u)\}]_{dB}$

(c) What is the relation between the two quantities above (i.e., which is larger, express one in terms of the other, etc).

7. Using the simulation programs that you developed for HW 1, problem 5, determine the outage probability for the worst case forward channel geometry. Create plots of $P_{O,CCI}(\text{SIR}_{Th})$ vs. SIR_{Th}, where SIR_{Th} is the threshold SIR value (i.e., outage occurs if $\text{SIR}(u) < \text{SIR}_{Th}$). Plot the y-axis on a linear scale and the x axis in dB. Produce plots for various cases considered in HW 1.

8. **Microeconomics** A particular macrocellular system, based on the GSM standard, has been designed to operate at $f_c = 900$ MHz with a 4/12 frequency reuse plan (i.e., $N_c = 4$ with 120 degree sectoring). It is desired to utilize this system in a microcellular system at a carrier of 2 GHz.

   It is known that the path loss exponent for macrocellular systems ranges from $\beta = 3$ to $\beta = 4$. For microcellular, the range is $\beta = 2$ to $\beta = 8$.

   (a) Based only on the path-loss model, what is the worst case (forward channel) C/I for the macrocellular system?

   (b) If the 4/12 reuse pattern is also used in the microcellular system, what is the worst case C/I?

   (c) In an effort to improve the C/I for the microcellular system, two different frequency reuse plans are considered:

   Plan 1: $N_c = 7$ with 120 degree sectoring

   Plan 2: $N_c = 4$ with 60 degree sectoring

   compute the worst case C/I for these two plans for the microcellular system:

   (d) With the standard receiver processing, the GMSK modulation used requires a C/I of at least 9.2 dB to provide the required BER. What is the C/I margin for each of the following: Macro, Micro(Plan 1), and Micro(Plan 2)?

   (e) It is also known that when considering log-normal shadowing, the shadow standard deviation $\sigma_{\epsilon(dB)}$ is larger for the microcellular environment than for the macrocellular system. Would you expect this to require a larger C/I margin or a smaller C/I margin for the microcellular system (i.e., relative to the macrocellular system)?

   (f) Suppose that the radius of the micro cell is 1/4 that of the macro cell, i.e., $R_{micro} = R_{macro}/4$ and that the area of a cell is well-approximated by πR^2. Assuming that the C/I values obtained in part (c) are tolerable, what is the increase in capacity associated with change to a micro-cellular plan. Specifically, specify the ratio of the maximum number of supportable users C_{micro}/C_{macro} for each of the two plans. Assume that the area of the service region is much greater than πR^2_{macro}.

9. It is proposed to build a cellular system for data communication based on the IS-95 standard. Specifically, the current system with a maximum vocoder rate of 9.6 Kbps and a spreading ratio of $\eta_v = 128$ will be modified by reducing the bit time sufficiently to provide 28.8 Kbps data services. The only proposed change to the system parameters is the change of the bit rate.

   (a) What is the spreading ratio for the proposed data system?

   (b) The current voice system can support K_v voice users. Because data services require a lower BER, the required E_b/N_0 for the proposed data system is 4 dB greater than that required for the current voice system. What is the approximate number of users that can be supported by the data-oriented system (i.e., K_d)? (Do not consider the effects of possible voice activity gains in this computation).

   (c) In the proposed data system, active users will be transmitting at 28.8 Kbps continuously in time (i.e., no bursty traffic). If the effects of voice activity are considered, will this reduce or increase the above estimate of K_d?

   (d) Based on these rough calculations (excluding voice activity effects), how much should a service provider charge for a data user relative to a voice user? How does this price ratio compare to the ratio of data rates?

10. **Multiple Options** A new mobile system standard is being developed. In an effort to take the best of both TDMA and CDMA, the designers of this system are considering using a hybrid TDMA/CDMA system. In such a system, the CDMA format replaces the FDMA aspect of systems like GSM and IS-54 (i.e., the RF channels).

    The total bandwidth for the system is 2.7 MHz (i.e., 2.7 MHz for the forward link and 2.7 MHz for the return link with frequency duplexing). The vocoder rate is 10 Kbps.

    Consider first the traditional TDMA/FDMA option. It has been determined that a reuse factor of 4 is required with 60 degree sectoring. The RF channels are 28 KHz each, with 3 TDMA slots per frame.

    (a) How many RF channels will be allocated to each sector of the TDMA/FDMA system?

    (b) How many users can be supported (maximum) per cluster?

    Now consider the TDMA/CDMA system. This option uses an 1/3 reuse pattern – i.e., the system bandwidth is partitioned using $N_c = 1$ reuse with 120 sectoring. Again, there are 3 TDMA slots per frame. However, in the TDMA/CDMA system there is one RF channel per sector (i.e., each active user uses the full bandwidth assigned to the sector). TDMA user signals are spread using direct sequence so that more than one user in the sector can use the same TDMA slot.

    (a) What is the bandwidth of the single RF channel per sector? What is the (approximate) spreading ratio associated with the spreading (do not consider the rate expansion due to TDMA slotting as part of the spreading)? Specifically, what is the spreading ratio which allows separation of active users sharing a given TDMA slot?

(b) If the CDMA format allows $K_s = \epsilon \eta_{C/T}$ users in a given sector to simultaneously share the same TDMA slot, how many users can be supported in a cluster? What value of ϵ would correspond to the two approaches being able to support the same number of users over a large geographical region? (assume that the reuse plan for the TDMA/CDMA system completely eliminates interference from other cells). Also, approximately what operating E_b/N_0 for the TDMA/CDMA system does this imply? (disregard voice activity).

11. Consider a 120-degree sectored system with a hexagonal reuse pattern. In lecture, we discussed the worst case C/I based on the path-loss model. This assumed the worst-case geometry and all potential first-tier interferers were active. Suppose, instead that we consider the worst case geometry, but consider a *traffic model* for the interferers.

    Specifically, assume that each interferer is active (i.e., that channel is being used in the other cell) with probability p and off with probability $(1-p)$ and that interferers act independently of each other. Assuming an inverse-power path-loss model, compute the average value of I/C for this system in terms of the worst-case I/C.

12. Recently, a new way to do error correction coding, "turbo" coding has been introduced. Turbo codes provide a better *coding gain* than the methods currently used. This means that the required E_b/N_0 in a turbo coded system is less than that of currently designed systems. For concreteness, we consider a system for which the new turbo codes provide an additional 4 dB of coding gain:

$$\left[\left(\frac{E_b}{N_0}\right)_{req,turbo}\right]_{dB} = \left[\left(\frac{E_b}{N_0}\right)_{req,current}\right]_{dB} - 4\text{dB}$$

    The current system is a CDMA-based system with an inverse-power path loss model $\beta = 4$ and log-normal shadowing $\sigma_{\epsilon_{dB}} = 8$ dB. The required operational BER is 10^{-4} and the system is designed so that the outage probability due to thermal noise at the boundary of the cell is less than 1%.

    (a) Without changing the quality of the system service, or the transmitter power, approximately how much larger could the cell area be made if the turbo code was used?

    (b) Assuming that the current system is heavily loaded, can an increase in system capacity be realized by using the new turbo codes? If so, give the approximate increase in the number of users K that could be supported. If not, explain why. Circle one answer and provide the additional details.

13. Let
$$P\left(\mathcal{E}\,|\Gamma(u)=\gamma\right)=Q\left(\sqrt{2\gamma}\right)$$
where $\Gamma(u)$ is a measure of the signal to noise ratio. Consider the case where
$$f_{\Gamma(u)}(\gamma)=\frac{1}{m_\gamma}e^{-\gamma/m_\gamma}, \qquad \gamma\geq 0$$
where $m_\gamma=\mathbb{E}\left\{\Gamma(u)\right\}$.
    (a) Show that
    $$P\left(\mathcal{E}\right)=\frac{1}{2}\left(1-\sqrt{\frac{m_\gamma}{1+m_\gamma}}\right)$$
    (b) Find
    $$\lim_{m_\gamma\to\infty}\frac{P\left(\mathcal{E}\right)}{1/m_\gamma}$$
    (c) Find $P(\mathcal{E})$ and repeat (b) for
    $$P\left(\mathcal{E}\,|\Gamma(u)=\gamma\right)=\frac{1}{2}e^{-\gamma}$$

    Note that this represents the bit error probability as a function of the average E_b/N_0 for Rayleigh fading (i.e., when $\sqrt{E_b(u)}$ is a Rayleigh random variable, $\Gamma(u)=E_b(u)/N_0$ is distributed as above). Part (a) represents BPSK and part (c) represents DBPSK.

14. Suppose that a BPSK modulation is used for transmitting information over an AWGN channel with $N_0/2=10^{-10}$ W/Hz. The transmitted signal energy is $E_b=A^2T/2$, where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-6} when the data rate is (a) 10 kbits/sec (kbps), (b) 100 kbps, (c) 1 Mbit/sec.

15. Plot the bit-error rate (BER) vs. E_b/N_o for the following coherent modulations
    (a) i. BPSK, DBPSK, QPSK, 2–orthogonal, 4–orthogonal.
        ii. 8PSK, 16PSK, 16QAM.
    (b) What is E_s/N_o for each of the above schemes?

16. For a QPSK constellation, shown in Fig. 1, a Gray encoder maps a binary bit stream to a QPSK (4-ary) symbol sequence through the rule given in Table I (i.e., $s_m=\sqrt{E_s}e^{j\frac{\pi}{2}m}$):

    Consider the following source bit stream:
    $$\{\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ \}$$

TABLE I
Gray mapping for QPSK

Source bits	Output symbols
00	s_0
01	s_1
10	s_3
11	s_2

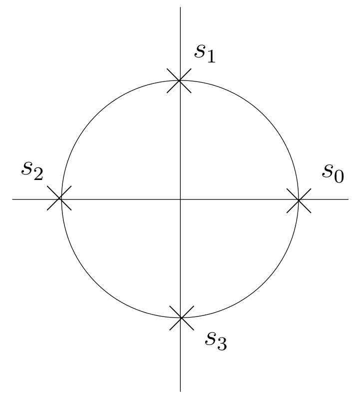


Figure 1: QPSK constellation

(a) Map this bit stream to a QPSK (4-ary) symbol stream using Gray encoding. Write down the sequence of encoded symbols.

(b) Differentially encode the bit stream to a QPSK symbol stream, assuming the initial transmit reference phase is $\phi_0 = 0$ degrees. Write down the sequence of phases of the encoded symbols in degrees.

(c) Map the bit stream to a $\pi/4$-DQPSK sequence. Write down the sequence of phases of the encoded symbols in degrees.

17. Show that MSK is a constant envelope modulation format. Determine the phase modulating signal.

18. Consider a multipath channel with equal power paths at delay 0 and delay Δ, where $\Delta < T$; T being the symbol time. If BPSK with root-raised cosine pulse shaping is used, then the output of the matched filter is modeled by

$$Z(u) = \sqrt{E_b} B_k(u) + \sqrt{E_b} \left[1 - R_p \left(\frac{\Delta}{T} \right) \right] B_{k-1}(u) + N(u)$$

where $B_k(u)$ and $B_{k-1}(u)$ are independent and each equally likely to be ± 1, $N(u)$ is Gaussian with zero mean and variance $N_o/2$, and $R_p(\tau)$ is the pulse correlation

$$R_p(\tau) = \text{sinc}(\tau) \frac{\cos(\beta \pi \tau)}{1 - (2\beta\tau)^2}$$

The factor β is the excess bandwidth of the pulse (i.e., bandwidth increases as β increases) and $\text{sinc}(x) = \sin(\pi x)/(\pi x)$.

(a) Find the bit error probability for a receiver deciding on $B_k(u)$ by comparing $Z(u)$ against zero. **Hint:** You have already solved an equivalent problem!

(b) Plot the symbol error probabilities as a function of E_b/N_0 for $\beta = 0, 0.5, 1$ for each of

$$\frac{\Delta}{T} = 0.0, 0.01, 0.1, 0.25, 0.5, 0.75$$

Note that for $\beta = 0$, the pulse correlation reduces to $R_p(\tau) = 1 - \tau$ – i.e., the pulse is rectangular.

(c) Now assume that your channel is fading, i.e., $\sqrt{E_b}$ is Rayleigh, and define the average SNR as

$$\mathbb{E}\left\{\frac{E_b}{N_0}\right\} = \frac{\bar{E}_b}{N_0}$$

Calculate and plot the symbol error probability as a function of the average SNR, $\bar{E}_b/N_0$, for the above values of Δ/T and β.

Hint: See Problem 1. For this, $m_\gamma = \bar{E}_b/N_0$, and use $N_0' = xN_0$ where x is determined from your solution to part (a) of this problem.

(d) Based on this example, what would you conclude about the role of the signal bandwidth with regard to robustness to unexpected multipath?

19. Consider the detailed effects of multiple access interference and multipath interference in a Direct Sequence CDMA system. Specifically, consider the case where the k^{th} user signal is defined by (complex baseband)

$$s^{(k)}(t; \mathbf{a}^{(k)}) = \sqrt{E^{(k)}} \sum_i a_i^{(k)} u_i^{(k)}(t - iT_s)$$

where the spreading and pulse shaping have been combinined into $u_i^{(k)}(t)$ as follows:

$$u_i^{(k)}(t) = \frac{1}{\sqrt{\eta}} \sum_{l=0}^{\eta-1} c_{i\eta+l}^{(k)} p(t - lT_c).$$

where $p(t)$ is the real-valued chip pulse which is assumed to be Nyquist relative to T_c – i.e., $p(t) * p(-t)|_{t=mT_c} = \delta_K(m)$. Assume tha the spreading ration $\eta = T_s/T_c$ is an integer.

(a) An important quantity is the following

$$R_{u_i u_j}^{(k,l)}(n) = u_i^{(k)}(\tau) * u_j^{(l)}(-\tau)\Big|_{\tau=nT_c} = \int_{-\infty}^{\infty} u_i^{(k)}(\tau + nT_c) u_j^{(l)}(\tau)\, d\tau.$$

Determine $R_{u_i u_j}^{(k,l)}(n)$ in terms of the partial period code cross-correlation

$$C^{(k,l)}(n; m_1; m_2) = \sum_{m=m_1}^{m_2} c_{m+n}^{(k)} c_m^{(l)}.$$

Hint: Consider three cases separately: (i) $|n| > \eta$, (ii) $0 \leq n < \eta$, (iii) $-\eta < n < 0$.

(b) Consider a random code model for the spreading sequences. Namely, consider the case when $c_n^{(k)}$ is an i.i.d. sequence of equally likely ± 1's. In this case, $R_{u_i u_j}^{(k,l)}(n)$ is a random sequnece. Show that

$$\mathbb{E}\left\{R_{u_i u_j}^{(k,l)}(n)\right\} = \delta_K(i-j)\delta_K(n)\delta_K(k-l)$$

$$\text{var}\left[R_{u_i u_j}^{(k,l)}(n)\right] = \begin{cases} 0 & (|n| \geq \eta) \text{ or } (i=j,\ k=l, \text{ and } n=0) \\ \frac{1}{\eta}\left[1-\frac{|n|}{\eta}\right] & \text{otherwise.} \end{cases}$$

The approximation of $R_{u_i u_j}^{(k,l)}(n)$ by this mean is called the *ideal code approximation*.

20. Assume that a DS signal $s(t;\mathbf{a})$ (as in the previous problem) is transmitted and the following (complex baseband) signal is received:

$$r(u,t) = y(t;\tilde{\mathbf{a}}_i) + w(u,t)$$

where $w(u,t)$ is the complex baseband equivalent of AWGN (i.e., zero mean, complex-circular with spectral level N_0) and

$$y(t;\tilde{\mathbf{a}}_i) = \sum_i a_i \sum_{m=0}^{L_g-1} g_m u_i(t-(i\eta+m)T_c)$$

where g_m represents the complex gain of a multipath at delay mT_c.

Consider the case when the receiver collects the following correlations:

$$r_i(m) = \int_{-\infty}^{\infty} r(t)u_i(t-(i\eta+m)T_c)\,dt = y_i(m)+w_i(m), \quad i=0,1,\dots I \quad m=0,1,\dots L_g-1$$

(a) Show that, using the ideal code assumption from the previous problem: (i) $y_i(m) = a_i g_m$ and that (ii) $w_i(m)$ is a white Gaussian sequence.

(b) If $a_i = \pm 1$ (BPSK modulation) and the receiver makes a decision via

$$\Re\left\{\sum_{m=0}^{L_g-1} g_m^* r_i(m)\right\} \underset{\mathcal{H}_{-1}}{\overset{\mathcal{H}_{+1}}{\gtrless}} 0.$$

Find the probability of bit error.

21. For a DS-CDMA system with the random code model developed in Problem 19, consider the effects of another CDMA user. Specifically, if user 1 is desired, and the receiver computes

$$r_i = \int_{-\infty}^{\infty} r(t)u_i^{(1)}(t-iT_s)dt$$

to decide on $a_i^{(1)}$, determine the impact of the presence of another user's signal – i.e.,

$$r(u,t) = s^{(1)}(t;\mathbf{a}^{(1)}) + s^{(2)}(t-\tau_2;\mathbf{a}^{(2)}) + w(u,t)$$

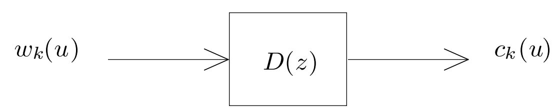
Specifically,

(a) Find the statistcial model for r_i of the form $r_i(u) = x_1 + x_2(u) + w(u)$. Assume that $\tau_2 = lT_c$, where l is an integer. What type of random variable is $x_2(u)$?

(b) Justify approximating $x_2(u)$ in the above as a Gaussian random variable. Given this approximation, $x_2(u) + w(u)$ can be considered a Gaussian random variable. What is the mean and variance of this effective noise? If BPSK modulation is used, what is the praobability of bit error?

.

22. Consider a one-pole auto-regressive filtered Gaussian noise approximation to the Clarke's isotropic spectrum. Specifically,



$$c_k(u) = \rho\, c_{k-1}(u) + \sqrt{1-\rho^2}\; w_k(u)$$
$$D(z) = \frac{\sqrt{1-\rho^2}}{1-\rho z^{-1}}$$

where $w_k(u)$ is a white complex Gaussian process with variance $\sigma^2 = 1$. This results in process $\{c_k(u)\}$ with power spectral density and correlation function

$$S_c(\nu) = \frac{1-\rho^2}{(1+\rho^2) - 2\rho\cos(2\pi\nu)}$$
$$R_c(m) = \rho^{|m|}$$

We are interested in simulating Clarke's spectrum with $\nu_D = 10^{-2}$.

(a) **Choosing ρ:** Consider three methods for choosing the value of ρ and determine ρ for each case:

i. $\nu_D = 3$ dB bandwidth: select ρ so that

$$S_c(\nu_D) = \frac{S_c(0)}{2}$$

ii. Correlation mapping: select ρ so that the one-symbol correlation agrees with Clarke's.

$$\rho = J_0(2\pi\nu_D)$$

where $J_0(\cdot)$ is the Bessel function of first kind and order zero.

iii. $\underline{\nu_D = 90\% \text{ bandwidth:}}$ Define the 90% power bandwidth, $B_{90\%}$, of the process $\{c_k(u)\}$ by

$$0.9R_c(0) = \int_{-B_{90\%}}^{B_{90\%}} S_c(\nu)d\nu$$

Select ρ so that $B_{90\%} = \nu_D$.

(b) Plot $R_c(m)$ for each of your three solutions along with the isotropic correlation $R_c(m) = J_0(2\pi\nu_D m)$.

(c) Plot $S_c(\nu)$ for each of your three solutions along with the Clarke's spectrum.

(d) Plot a realization of N=1000 points of the process $c_k(u)$ for each solution with a realization of Clarke's isotropic process. Specifically, plot $20\log_{10}|c_k(u)|$ vs. k.

Notes:

i. Let the system come to a steady-state before plotting.

ii. A file with N=1000 points of realization of isotropic process will be available at the class homepage.

23. Consider a $D = 2$ correlated diversity system, denoted by

$$\mathbf{z}(u) = \mathbf{c}(u)\sqrt{E}A(u) + \mathbf{w}(u)$$

where $\mathbf{z}$, $\mathbf{w}$, and $\mathbf{c}$ are (2×1) complex vectors and $A(u) \in \{-1, +1\}$ is the data bit (equally likely to take either value). The relevant statistics are:

$$\begin{aligned}
\mathbf{m_w} &= \mathbf{0} \\
\mathbf{K_w} &= \mathbb{E}\left\{\mathbf{w}(u)\mathbf{w}^\dagger(u)\right\} = N_0\mathbf{I} \\
\mathbf{m_c} &= \mathbf{0} \\
\mathbf{K_c} &= \mathbb{E}\left\{\mathbf{c}(u)\mathbf{c}^\dagger(u)\right\} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}
\end{aligned}$$

and both $\mathbf{w}(u)$ and $\mathbf{c}(u)$ are circular (e.g., $\mathbb{E}\{\mathbf{w}(u)\mathbf{w}^t(u)\} = \mathbf{O}$).

(a) With knowledge of $\mathbf{c}$, the MRC receiver makes a decision according to

$$\Re\left\{\mathbf{c}^\dagger\mathbf{z}(u)\right\} \underset{\mathcal{H}_{-1}}{\overset{\mathcal{H}_{+1}}{\gtrless}} 0$$

Show that, given $\mathbf{c}(\mathbf{u}) = \mathbf{c}$, the probability of error is given by

$$P(\mathcal{E}|\mathbf{c}) = \mathrm{Q}\left(\sqrt{\frac{2E}{N_0}\|\mathbf{c}\|^2}\right)$$

Hint: $\Re\{x\}\Re\{y\} = \Re\{xy^* + xy\}/2$.

(b) Using a computer program, evaluate the average performance for the MRC system with $\rho = 0, 0.1, 0.2, 0.5, 0.6, 0.9$.

(c) Given $\mathbf{c}$, the selective combiner makes a decision by

$$\Re\left\{c_{max}^* z_{max}(u)\right\} \underset{\mathcal{H}_{-1}}{\overset{\mathcal{H}_{+1}}{\gtrless}} 0$$

where $z_{max}(u)$ is the component of $\mathbf{z}(u)$ with largest $|c_n|$, $n = 1, 2$ and c_{max} is the corresponding gain. Show that, given c_{max}, the probability of error is given by

$$P(\mathcal{E}|c_{max}) = \mathrm{Q}\left(\sqrt{\frac{2E}{N_0}|c_{max}|^2}\right)$$

(d) Using a computer program, evaluate the average performance for the a selective combining system with $\rho = 0, 0.1, 0.2, 0.5, 0.6, 0.9$.

24. This problem illustrates the difference between MRC, selective combining, and no diversity. Consider the case where a diversity branch gain $h^{(d)}(u)$ is a complex, circular Gaussian process with $E_d(u) = |h^{(d)}(u)|^2$. Then, $E_d(u)$ is an exponential random variable with mean γ_d.

First, consider a system with no diversity. This has average energy $\bar{E} = \mathbb{E}\left\{E(u)\right\} = \gamma$, with variance $\sigma^2 = \mathrm{var}\left[E(u)\right] = \gamma^2$.

Next, consider a system with D orders of diversity, each having $\mathbb{E}\left\{E_d(u)\right\} = \gamma_d = \gamma/D$. Then, with MRC, the effective average energy is $\bar{E}_{MRC} = \gamma$ and variance $\sigma^2_{MRC} = \gamma^2/D^2$. It can be shown that, for this same system with selective combining,

$$\bar{E}_S = \gamma\left[\frac{1}{D}\sum_{n=1}^{D}\frac{1}{n}\right]$$

$$\sigma_S^2 = \gamma^2\left[\frac{1}{D^2}\sum_{n=1}^{D}\frac{1}{n^2}\right]$$

Plot $\bar{E}$ and σ^2 for the MRC and selective cases normalized by the no-diversity counterparts for $D = 2, 3, 5, 20$. Discuss your results.

25. A new digital modulation technique is proposed based on advanced new wavelet signals. Given a bit-stream $a_k \in \{0, 1\}$, the transmitted waveform has the form

$$S(t; \mathbf{a}) = \sqrt{E}\sum_k \left[(1 - a_k)w_0(t - kT) + a_k w_1(t - kT)\right]\sqrt{2}\cos(2\pi f_c t)$$

Here, $w_0(t)$ and $w_1(t)$ are sophisticated wavelet shape functions localized in time such that $w_0(t) = 0$ for $t \notin [0, T_b/2)$ and $w_1(t) = 0$ for $t \notin [T_b/2, T_b)$. They are both real-valued and normalized so that $\int w_i^2(t)dt = 1$ for $i = 0, 1$.

(a) If this WPM scheme is used in an AWGN channel with noise spectral level $N_0/2$ and coherent detection is performed, sketch the block diagram of the best receiver. What is the probability of bit error for this receiver?

(b) Can this modulation be detected in a noncoherent manner? If so, give the probability of error with noncoherent detection. If not, explain why.

26. The forward link of a particular mobile cellular system uses uncoded BPSK modulation. The channel is well modeled by a $\beta = 4$ path-loss model, log-normal shadowing with shadowing deviation $\sigma_{\epsilon_{dB}} = 6$, and flat Rayleigh fading. The short-term Rayleigh fading has the Clarke spectrum with a normalized Doppler spread of $\nu_D = 0.005$. You may assume that the receiver perfectly estimates and tracks the channel conditions.

    (a) Outage for this system is defined as a bit error rate (BER) larger than 0.01. What is the minimum required short-term averaged bit-energy to noise spectral level required to avoid outage? Specifically, if $\bar{E}_b/N_0 > \left[\bar{E}_b/N_0\right]_{req}$ implies no outage, what is $\left[\bar{E}_b/N_0\right]_{req}$ in dB?

    (b) Assume negligible CCI and an outage requirement due to thermal noise of less than 5% at the cell boundary – i.e., $P_{out,N_0}(R) < 0.05$. What is the required power margin M over $\left[\bar{E}_b/N_0\right]_{req,dB}$ in dB required to meet this outage requirement?

    (c) Given the design choices resulting from (a) and (b), consider a mobile unit operating at half the cell radius from the base-station – i.e., $d = R/2$. What is the nominal BER of this mobile unit? More specifically, what is the BER of the mobile when it operates at the nominal long-term SNR (i.e., no shadow effects)?

27. For this problem consider a Rayleigh fading system with D orders of uncorrelated, explicit diversity which are combined using Maximum ratio Combining (MRC).

    (a) Consider a plot of $y = \log_{10}(P_b)$ vs. $x = \left[\bar{E}_b/N_0\right]_{dB}$. For large values of x, the slope of the curve is roughly constant at S. Find an expression for D in terms of the slope S.

    (b) Estimate the effective order of diversity for the system with the BER vs. $\bar{E}_b/N_0$ shown below.

28. After matched-filtering on an ideal AWGN channel, the effects of a carrier phase estimation error can be modeled by

$$Z(u) = \sqrt{E_b}B(u)\cos\Phi(u) + W(u)$$

where $B(u)$ is equally likely to be $+1$ or -1, $W(u)$ is a real Gaussian random variable with zero mean and variance $N_0/2$, and $\Phi(u)$ is the phase error between the actual carrier phase and the receiver's phase estimate. This phase estimation error is modeled as a random variable. Due to modes in the reference oscillator, this is modeled as a discrete random variable with probability mass function:

$$P(\Phi(u) = \phi) = \begin{cases} \frac{1}{16} & \phi = \pi/4, -\pi/4 \\ \frac{1}{4} & \phi = \pi/8, -\pi/8 \\ \frac{3}{8} & \phi = 0 \end{cases}$$

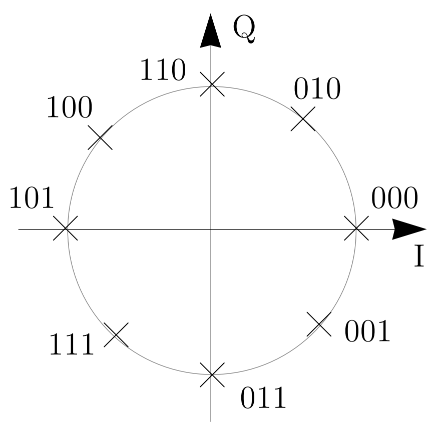
(a) Conditioned on $\Phi(u) = \phi$, find the probability of error $P(\mathcal{E}|\phi) = \Pr\left\{\hat{B}(u) \neq B(u)\right\}$ when the decision rule is $\hat{B}(u) = +1 \iff Z > 0$.

(b) Find the unconditional probability of error.

(c) For large E_b/N_0, what is the effective degradation in E_b/N_0 caused by the phase jitter?

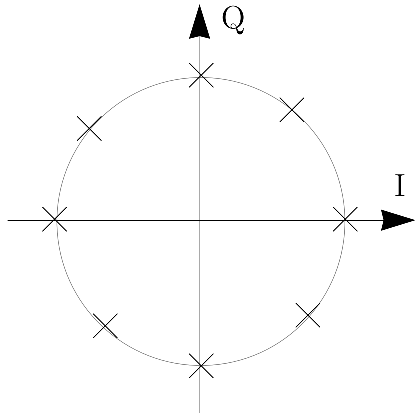
If DBPSK is used with "differentially coherent" detection, what is the unconditional error probability?

29. You are tasked to define a new modulation format: $\pi/8$-D8PSK. In this format, information bits are grouped in to triplets and mapped onto one of eight information phases according to the following Gray-map:



The modulation format is defined in a similar way to $\pi/4$-DQPSK. First, the bits are mapped onto a sequence of angles α_k according to the above mapping. Then, these angles are differentially encoded. The differentially encoded phases are then transmitted with a $\pi/8$ shift in the constellation inserted at each symbol time. The overall phase transmitted at symbol time k is β_k. You may assume that $\beta_0 = 0$.

(a) Modify the constellation below as necessary to indicate all the possible phases that may be transmitted (i.e., all possible values for β_k)



(b) Assuming that $\beta_{k-1} = 0$, draw all the possible phase transitions ($\beta_{k-1} \rightarrow \beta_k$).

(c) Complete the table below indicating the rule for mapping bit-triplets to phase differences ($\Delta\beta_k = \beta_k - \beta_{k-1}$) – numerical answers:

bit triplet	Transmitted phase differential: $\Delta\beta_k$
000	
010	
110	
100	
101	
111	
011	
001	

30. A particular mobile radio system is providing voice services at 10 Kbits/sec. The system uses QPSK modulation with a bandwidth of $W = 1/(2T)$, where T is the symbol time. The delay spread of the channel is 15% of the symbol time. Through simulation, it has been concluded that a delay spread of less than 25% of the symbol time can be modeled as flat fading with no ISI.

It is desired to upgrade this system to provide Internet access at roughly 30 Kbits/sec. As a system engineer, you have been asked to evaluate two options for implementing this upgrade

OPTION A: Maintain the bandwidth allocation as is (i.e., do not change the symbol time) and use MPSK modulation to improve the data rate.

OPTION B: Maintain the QPSK format currently used and decrease the symbol time from $T_{current}$ to T_{new}.

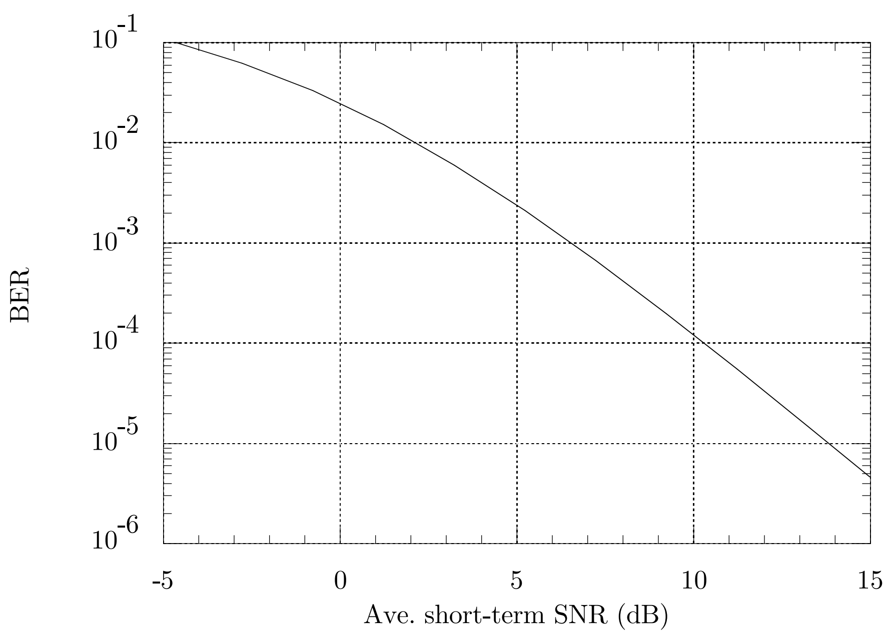
(a) For **OPTION A**, what is the value of M required to provide the upgrade in services?

(b) For **OPTION A**, what would you report as the biggest drawback of this approach – i.e., in what way does this change impact the current system design?

(c) For **OPTION B**, what is the value of T_{new} required to provide the upgrade in services?

(d) For **OPTION B**, what would you report as the biggest drawback of this approach – i.e., in what way does this change impact the current system design? Note: your boss already knows that this option will require the leasing of additional bandwidth, so do not report that.

31. A particular mobile radio system channel is modeled by an exponential path-loss model with $\beta = 3.5$ and log-normal shadowing with $\sigma_{\epsilon(dB)} = 6$. The short-term fading model is a slow, flat Rayleigh channel with three orders of diversity; this translates to the bit-error rate (BER) vs. average short-term SNR ($\bar{\gamma}$) curve as given below.



The system must provide a quality of service (QoS) defined by a BER performance better than 10^{-3}.

(a) The *threshold SNR* $\bar{\gamma}_T$ is the minimum value of the average short-term SNR that yields acceptable performance. Determine $\bar{\gamma}_T$

(b) The power at a reference point results in a long-term average SNR of $SNR_L =$ 43 dB. What is the long-term SNR at a point five (5) times as far from the transmitter? You may assume that the noise level N_0 is not a function of location.

(c) At the point of interest (i.e., 5 times the reference distance) what is the probability that the long-term SNR will not be sufficient for the required quality of service?

32. Consider a system with reuse $N_c = 7$ and no sectoring. The system undergoes log-normal shadowing. Recall that a log-normal random variable is one of the form

$$P(u) = 10^{\frac{1}{10}\left(\bar{P}_{dB} + \epsilon_{dB}(u)\right)}$$

where $\epsilon_{dB}(u)$ is Gaussian with zero-mean and standard deviation $\sigma_{\epsilon_{dB}}$. It has been determined that the cumulative effect of the co-channel interferers can be modeled as a log-normal random variable with $\bar{P}_{CCI,dB}$ and $\epsilon_{CCI,dB}(u)$.

If the effects of shadowing are ignored, there is a margin of $M_{CCI,dB}$ in SIR_{dB} relative to the minimum required value of SIR_{dB}.

(a) Determine the SIR in dB

(b) Denoting the shadowing deviation by $\sigma_{\epsilon_{S,dB}}$ and the standard deviation of $\epsilon_{CCI,dB}(u)$ by $\sigma_{\epsilon_{CCI,dB}}$, determine the probability of outage due to CCI.

(c) For the specific case of $M_{CCI,dB} = 10$, $\sigma_{\epsilon_{S,dB}} = 8$, and $\sigma_{\epsilon_{CCI,dB}} = 4$, determine the probability of outage due to CCI.

33. Consider a hybrid FDMA/TDMA system – i.e., with a design similar to that of GSM and IS-54. The entire system has 105.84 MHz of bandwidth for each the forward and return link (i.e., frequency duplexed). A reuse pattern of $N_c = 4$ and 120 degree sectoring has been selected. Each RF channel is 210 KHz and is shared by 7 users via TDMA – i.e., 7 TDMA slots per frame. While the vocoder is 10 Kbps, after adding error correction and packet overhead bits, each user must achieve a throughput of 20 Kbps on the channel. For this purpose a BPSK modulation format with root-raised cosine pulse shaping is used.

From this information, fill in the following table of system parameters:

System parameter	*(fill-in numerical value)*
RF channels per cluster	
RF channels per cell	
Max. number of active users per sector	
Burst rate	
RRC pulse excess bandwidth (β)	

34. Consider a CDMA system with two classes of users: high-priority users and low-priority users. Each priority communicates using the same bite-rate, modulation coding, etc.; with the only difference being that high-priority users are allowed a higher power. Specifically, if user i is a high-priority user, then the power control system sets $E_i = E_{high}$, otherwise, it sets $E_i = E_{low}$.

For concreteness, consider the case where $E_{high} = 2E_{low}$ and assume that 20% of the users are high-priority.

(a) Assume that the system is heavily loaded, the spreading ratio is η, and there is no voice activity gain. Also, denote the required E_b/N_0 for low-priority users by $(E_b/N_0)_{min,LP}$. Find an expression for the maximum number of total users.

(b) Suppose $\eta = 128$ and $(E_b/N_0)_{min,LP}$ corresponds to a BER of 10^{-3} with uncoded BPSK for the low-priority users. Determine the maximum number of total users

Determine the BER of the high-priority system when the system is fully loaded:

35. Recently, a new way to do error correction coding, "turbo" coding has been introduced. Turbo codes provide a better *coding gain* than the methods currently used. This means that the required E_b/N_0 in a turbo coded system is less than that of currently designed systems. For concreteness, we consider a system for which the new turbo codes provide an additional 4 dB of coding gain:

$$\left[\left(\frac{E_b}{N_0}\right)_{req,turbo}\right]_{dB} = \left[\left(\frac{E_b}{N_0}\right)_{req,current}\right]_{dB} - 4\text{dB}$$

The current system is a CDMA-based system with an exponential path loss model $\beta = 4$ and log-normal shadowing $\sigma_{\epsilon_{dB}} = 8$ dB. The required operational BER is 10^{-4} and the system is designed so that the outage probability due to thermal noise at the boundary of the cell is less than 1%.

(a) Without changing the quality of the system service, or the transmitter power, approximately how much larger could the cell area be made if the turbo code was used? Specifically, determine the ratio of the cell area for the new and current systems, $\frac{A_{cell,turbo}}{A_{cell,current}}$.

(b) Assuming that the current system is heavily loaded, can an increase in system capacity be realized by using the new turbo codes? If so, give the approximate increase in the number of users K that could be supported. If not, explain why.

36. Consider a 120-degree sectored system with a hexagonal reuse pattern. In lecture, we discussed the worst case C/I based on the path-loss model. This assumed the worst-case geometry and all potential first-tier interferers were active. Suppose, instead that we consider the worst case geometry, but consider a *traffic model* for the interferers.

Specifically, assume that each interferer is active (i.e., that channel is being used in the other cell) with probability p and off with probability $(1 - p)$ and that interferers act independently of each other.

Assuming an exponential path-loss model, compute the average value of I/C for this system in terms of the worst-case I/C.

37. A new digital modulation technique is proposed based on advanced new wavelet signals. Given a bit-stream $a_k \in \{0,1\}$, the transmitted waveform has the form

$$S(t;\mathbf{a}) = \sqrt{E}\sum_k \left[(1-a_k)w_0(t-kT) + a_k w_1(t-kT)\right]\sqrt{2}\cos(2\pi f_c t)$$

Here, $w_0(t)$ and $w_1(t)$ are sophisticated wavelet shape functions localized in time such that $w_0(t) = 0$ for $t \notin [0, T_b/2)$ and $w_1(t) = 0$ for $t \notin [T_b/2, T_b)$. They are both real-valued and normalized so that $\int w_i^2(t)dt = 1$ for $i = 0, 1$.

   (a) If this WPM scheme is used in an AWGN channel with noise spectral level $N_0/2$ and coherent detection is performed, sketch the block diagram of the best receiver. What is the probability of bit error for this receiver?

   (b) Can this modulation be detected in a noncoherent manner? If so, give the probability of error with noncoherent detection. If not, explain why.

38. Consider a system with reuse $N_c = 7$ and no sectoring. The system undergoes log-normal shadowing. Recall that a log-normal random variable is one of the form

$$P(u) = 10^{\frac{1}{10}\left(\bar{P}_{dB}+\epsilon_{dB}(u)\right)}$$

where $\epsilon_{dB}(u)$ is Gaussian with zero-mean and standard deviation $\sigma_{\epsilon_{dB}}$. It has been determined that the cumulative effect of the co-channel interferers can be modeled as a log-normal random variable with $\bar{P}_{CCI,dB}$ and $\epsilon_{CCI,dB}(u)$.

If the effects of shadowing are ignored, there is a margin of $M_{CCI,dB}$ in SIR_{dB} relative to the minimum required value of SIR_{dB}.

   (a) Determine the SIR in dB.

   (b) Denoting the shadowing deviation by $\sigma_{\epsilon_{S,dB}}$ and the standard deviation of $\epsilon_{CCI,dB}(u)$ by $\sigma_{\epsilon_{CCI,dB}}$, determine the probability of outage due to CCI.

   (c) For the specific case of $M_{CCI,dB} = 10$, $\sigma_{\epsilon_{S,dB}} = 8$, and $\sigma_{\epsilon_{CCI,dB}} = 4$, determine the numerical probability of outage due to CCI.

39. Consider a hybrid FDMA/TDMA system – i.e., with a design similar to that of GSM and IS-54. The entire system has 52.92 MHz of bandwidth for each the forward and return link (i.e., frequency duplexed). A reuse pattern of $N_c = 4$ and 120 degree sectoring has been selected. Each RF channel is 105 KHz and is shared by 7 users via TDMA – i.e., 7 TDMA slots per frame. While the vocoder is 10 Kbps, after adding error correction and packet overhead bits, each user must achieve a throughput of 20 Kbps on the channel. For this purpose a BPSK modulation format with root-raised cosine pulse shaping is used.

From this information, fill in the following table of system parameters

System parameter	*(fill-in numerical value)*
RF channels per cluster	
RF channels per cell	
Max. number of active users per sector	
Burst rate	
RRC pulse excess bandwidth (β)	

40. Consider a CDMA system with two classes of users: high-priority users and low-priority users. Each priority communicates using the same bite-rate, modulation coding, etc.; with the only difference being that high-priority users are allowed a higher power. Specifically, if user i is a high-priority user, then the power control system sets $E_i = E_{high}$, otherwise, it sets $E_i = E_{low}$.

    For concreteness, consider the case where $E_{high} = 2E_{low}$ and assume that 20% of the users are high-priority.

    (a) Assume that the system is heavily loaded, the spreading ratio is η, and there is no voice activity gain. Also, denote the required E_b/N_0 for low-priority users by $(E_b/N_0)_{min,LP}$. Find an expression for the maximum number of total users.

    (b) Suppose $\eta = 128$ and $(E_b/N_0)_{min,LP}$ corresponds to a BER of 10^{-3} with uncoded BPSK for the low-priority users. Determine the maximum number of total users. Determine the BER of the high-priority system when the system is fully loaded

41. An existing cell-based mobile system design uses uncoded BPSK modulation. This was selected in 1984 in an effort to simplify the handset processing. Due to relatively poor performance, the system has enjoyed limited success in the marketplace. A new start-up company, dBs.com, proposes a modification to the standard that uses coding and interleaving. Based on their design enhancement, the dBs.com CEO claims that significant coverage gains are possible making the languishing standard viable in rural areas.

    The desired (decoded) bit error rate (BER) is 10^{-4}. Based on simulations, the dBs.com team claims that their system has a 5 dB coding gain relative to the existing system at a BER of 0.01. Furthermore, through the time diversity, the dBs.com system has an effective diversity of 3, whereas the existing system has Rayleigh fading with a diversity of 1.

The large-scale fading model is an exponential path-loss model with $\beta = 3$ and log-normal shadowing with a 6 dB shadowing deviation. The outage rate due to thermal noise on the cell edge must be less than 5%.

You have just been hired to "solidify" the claims of the dBs.com team and need to act quickly!

(a) Based on the above information and your knowledge of fading channel performance, you estimate the coding gain of the system at the desired BER of 10^{-4}. What is this gain? Explain your work to your CEO.

(b) What is the improvement in coverage due to the dBs.com technology? Specifically, what is the ratio of the cell area of the redesigned system the original system?

42. A cell-based mobile communication system is being designed using the FDMA/TDMA approach similar to that of IS-54 and GSM. The total system bandwidth is 25 MHz (each in the forward and reverse link of a FDD system). The system will use a reuse 4, 120 degree sectoring design.

    Based on a preliminary field trial, the channel is known to have a maximum delay spread of 12 μ-sec and a coherence time of 50 msec.

    The vocoding, error-correction coding, and modulation have all been fixed. Specifically, the system uses 10 kbps vocoder, a rate 1/2, binary convolutional code with QPSK modulation, and root-raised cosine pulse shaping with roll-off factor 0.25.

    The remaining task is to design a suitable TDMA frame structure and assess the impact on the system capacity.

    (a) It is desired to select the number of TDMA slots per frame, N_{slot}, so that an equalizer is not needed. It has been determined that, as long as the delay spread is less than 1/3 of the channel (burst) symbol time, equalization can be avoided. Determine the maximum value of N_{slot} and the corresponding symbol time during each TDMA burst, T_s.

    (b) A training sequence is inserted at the start of each TDMA slot. To avoid adaptive receiver processing, the duration of the data burst following this training cannot be more than 5% of the coherence time. Based on the design in (a), determine the maximum length of the TDMA slot – i.e., max. TDMA slot duration in msec and max. number of vocoder bits/TDMA slot.

    (c) Given the above design choices, determine the bandwidth of each (FDMA) RF channel, and the maximum number of active users in a sector.

43. If $Y(u)$ is a Gaussian random variable with mean zero and variance 6, then determine $10\log_{10}\left[\mathbb{E}\left\{10^{(9+X(u))/10}\right\}\right]$

44. A 16 PSK modulation format is

   (a) More suitable for a TDMA system than a CDMA system
   (b) More suitable for a CDMA system than a TDMA system
   (c) Equally suitable to each.

45. **True or False:** Given a fixed C/I as predicted by a path loss model, after considering the impact of log-normal shadowing, if the interference comes form many users less shadowing margin will be required than if the interference is created by a single interferer.

46. **True or False:** An IS-54 based system with reuse 7 has 357 voice channel available per cell. Hence, care must be taken to ensure that only 357 cell phones are sold in that region

47. For fixed path-loss model, required BER, and outage probability, an increase in the modeled shadowing deviation with require

    (a) An increase in the transmitted power (more margin).
    (b) Less user mobility
    (c) More robustness to CCI

48. **True or False:** Staggered QPSK is preferred over QPSK because of its improved performance in an AWGN channel

49. Name two advantages of a CDMA system over a TDMA/FDMA system.

50. **True or False:** The antenna gain pattern will affect the shape of the Doppler spectrum experienced by a mobile unit in a dense scattering environment.

51. **True or False:** The most effective way to obtain practical time diversity is to retransmit the same information after waiting t_c seconds, where t_c is the channel coherence time.

52. If a synchronous CDMA system uses only Walsh functions for all spreading and multiple access, then is a standard Rake receiver applicable?

53. Soft hand-off in a CDMA system requires

    (a) Use of a Rake receiver
    (b) Artificial multipath delay of 1 to 3 chip times
    (c) Nonlinear dynamic programming

54. **True or False:** Coherent modulation formats are more applicable to slow frequency hopping systems than to fast frequency hopping systems

55. **True or False:** Decision Feedback Equalization typically works well for mobile fading channels and is the method utilized in most modern TDMA cellular/PCS transceivers.

56. A midamble is used in GSM instead of a preamble to

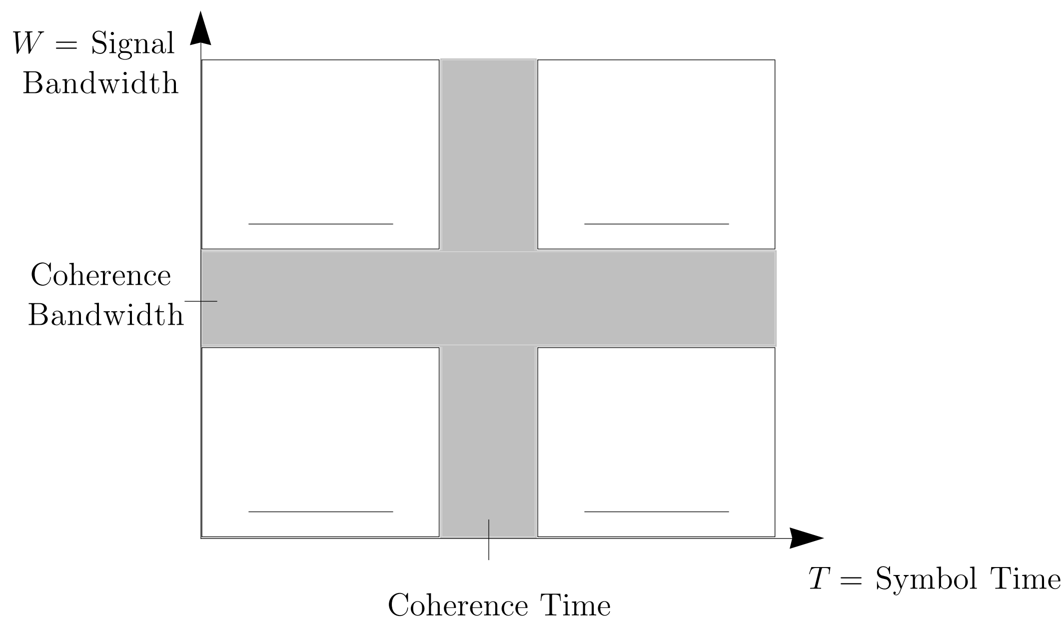
    (a) Reduce the amount of training overhead required

    (b) Reduce guard bands between TDMA slots

    (c) Improve the effective interleaver depth given the latency constraint

57. **True or False:** Drawing upon Shannon's separation theorem, the GSM source and channel coders were designed independently.

58. For a channel with fixed delay spread and coherence bandwidth, characterize the channel as *slow, fast, frequency-selective, or flat* for different regions of the signal parameters T (symbol time) and W (signal bandwidth). Specifically, in each blank region of the figure below write all of these terms that are applicable to that region – i.e., write *slow, fast, frequency-selective, and/or flat* as applicable.



59. **True or False:** Most commercial direct sequence systems are "short code" systems due to the lack of jamming threats.

60. **True or False:** The term "Rayleigh fading" arises since the in-phase and quadrature amplitudes are independent Rayleigh random variables.

61. Frequency diversity can be obtained

(a) Only in a spread spectrum system.
(b) In any system with frequency-selective fading.
(c) When the latency constraints allow sufficient interleaving.

62. In GMSK the "BT" factor effects the spectral roll-off as

    (a) Smaller BT means that the sidelobes roll off more quickly.
    (b) Larger BT means that the sidelobes roll off more quickly.
    (c) The choice of BT does not affect spectral roll-off.

63. Low-rate error correcting codes are

    (a) Equally appropriate for CDMA and TDMA systems
    (b) Most appropriate for TDMA systems.
    (c) Most appropriate for CDMA systems.

64. The coherence time of the channel is

    (a) Independent of the carrier frequency.
    (b) Roughly unaffected by the user speed.
    (c) Inversely proportional to the Doppler spread.

65. **True or False:** Co-channel interference in a TDMA system is well modeled as additional AWGN for the purposes of system capacity calculations.

66. **True or False:** In addition to the improved bandwidth efficiency, IS-54 has a trunking efficiency gain relative to AMPS.

67. The shape of the pulse used in a general QAM system is important since it

    (a) Determines the bit error rate of the system.
    (b) Determines the efficiency of the coding used.
    (c) Impacts the frequency spectrum of the transmitted signal.

68. **True or False:** Effective communication at rates above the channel capacity is possible if offset modulation techniques are combined with frequency-hopping.

69. **True or False:** One advantage of modulation formats with memory is that the receiver complexity is reduced relative to that for memoryless modulation techniques.

70. **True or False:** Slow frequency-hopping techniques are more compatible with BPSK than fast frequency hopping techniques

71. **True or False:** The term "Rayleigh Fading" refers to the statistical properties of the coherence time.

72. **True or False:** In-phase and Quadrature channels are realized in practice by narrow-band modulation of two carriers which are 90 degrees out of phase.

73. The period of a PN sequence generally impacts

    (a) The spreading ratio.
    (b) The need for temporal guard-bands.
    (c) The time required for the receiver to obtain synchronization information.

74. Noncoherent detection may be considered for a system because

    (a) It eliminates the need for costly phase-tracking loops.
    (b) It allows the use of non-linear amplifiers.
    (c) Helps mitigate the effects of log-normal shadowing.

75. **True or False:** Differential BPSK outperforms the same BPSK system without differential encoding and differentially coherent detection on an ideal AWGN system.

76. **True or False:** Offset modulation formats are useful in practical systems because they have good spectral properties for systems with filtering and nonlinearities.

77. **True or False:** GMSK is obtained from MSK by shaping the amplitude.

78. **True or False:** Walsh functions provide examples of maximum length shift register sequences (i.e., PN sequences)

79. The CDMA digital cellular standard IS-95 is based on

    (a) Direct sequence
    (b) Frequency-hopping
    (c) Time-hopping

80. In a digital communication transmitter, which operation is performed first?

    (a) Error correction coding (channel coding)
    (b) Modulation
    (c) Compression (source coding)

81. The constellation is a useful tool for characterizing a general QAM format because it

    (a) Provides the error performance curve by inspection.

(b) Describes the possible signals that can be sent during a symbol time.

(c) Emphasizes the importance of the Doppler spread of the channel.

82. **True or False:** The coherence time for Clarke's isotropic model is on the order of the time required to travel 25 wavelengths.

83. The performance of a spread spectrum system over a single-user, ideal AWGN channel

(a) Decays inverse-linearly with the spreading ratio η.

(b) Depends heavily on the PN code used.

(c) Is the same as the corresponding system without spreading.

84. **True or False:** Clarke's isotropic model is only one of many ways to model the delay spread of a channel

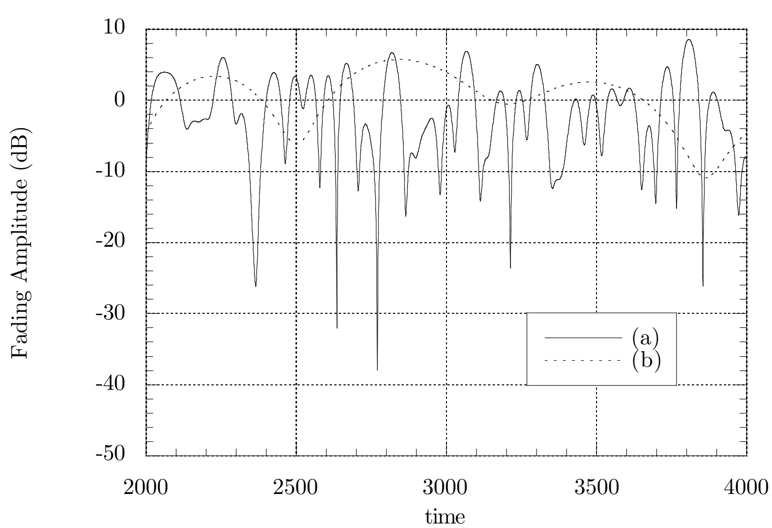
85. **True or False:** With current technology, frequency-hopping techniques are capable of larger bandwidth expansion than direct-sequence techniques.

86. Referring to the two plots below showing the amplitude of realizations of a Clarke's isotropic process under different dynamics, the Doppler spread is larger for

(a) The channel corresponding to (a)

(b) The channel corresponding to (b)

(c) There is not enough information to determine this with any confidence.



87. **True or False:** The IS-95 CDMA system uses a bandwidth efficient modulation based on continuous phase modulation for the return link.

88. Name two practical factors that prevent the IS-95 CDMA system from achieving the ideal capacity of approximately 20 times that of AMPS:

89. **True or False:** Multipath effects introduce multiple access interference on the forward link of the IS-95 system despite the orthogonal signature sequences.

90. A given FDMA cellular system has been designed. Reuse factors of $N_c = 4$ and $N_c = 7$ are under consideration. For a fixed channel assignment scheme and the same grade of service requirement in each case, answer the following by circling the correct answer:

    (a) Which design supports a larger maximum number of active subscribers per cluster?

    (b) Which design supports a larger maximum number of subscribers (i.e., not necessarily active) per cluster?

91. **True or False:** Root raised cosine pulses are generally preferred over square pulse shapes because they provide performance improvements equivalent to between 0.5 to 1.5 dB of SNR.

92. Consider a system with reuse factor $N_c > 1$. Holding all other parameters fixed, indicate how the overall system capacity varies with:

Increasing cell area:	*capacity*	increases	decreases	no change
Decreasing reuse factor (N_c):	*capacity*	increases	decreases	no change
Adding sectoring:	*capacity*	increases	decreases	no change
Larger path loss exponent (β):	*capacity*	increases	decreases	no change

93. A system operates with a power margin of 6 dB for outage due to thermal noise at the cell edge. If the maximum outage probability at the cell edge is 0.01, what is the maximum shadowing deviation that can be tolerated?

94. The pulse shape defined by

$$p(t) = \begin{cases} e^{-t/T} & 0 \le t < T \\ 0 & \text{otherwise} \end{cases}$$

satisfies the Nyquist criterion.

95. Consider a system with reuse factor $N_c > 1$. Holding all other parameters fixed, indicate how the SIR due to CCI (worst case forward link) varies with:

Increasing reuse factor (N_c):	*SIR*	increases	decreases	no change
Adding sectoring:	*SIR*	increases	decreases	no change
Decrease cell radius:	*SIR*	increases	decreases	no change
Larger path loss exponent (β):	*SIR*	increases	decreases	no change

96. **True or False:** The IS-95 CDMA system uses a bandwidth efficient modulation based on continuous phase modulation for the return link.

97. Name two practical factors that prevent the IS-95 CDMA system from achieving the ideal capacity of approximately 20 times that of AMPS.

98. **True or False:** Multipath effects introduce multiple access interference on the forward link of the IS-95 system despite the orthogonal signature sequences.

99. A given FDMA cellular system has been designed. Reuse factors of $N_c = 4$ and $N_c = 7$ are under consideration. For a fixed channel assignment scheme and the same grade of service requirement in each case, answer the following:

    (a) Which design supports a larger maximum number of active subscribers per cluster? ($N_c = 4$, $N_c = 7$, or both are the same)

    (b) Which design supports a larger maximum number of subscribers (i.e., not necessarily active) per cluster? ($N_c = 4$, $N_c = 7$, or both are the same)

100. **True or False:** Root raised cosine pulses are generally preferred over square pulse shapes because they provide performance improvements equivalent to between 0.5 to 1.5 dB of SNR.

101. Consider a system with reuse factor $N_c > 1$. Holding all other parameters fixed, indicate how the overall system capacity varies with:

Increasing cell area:	*capacity*	increases	decreases	no change
Decreasing reuse factor (N_c):	*capacity*	increases	decreases	no change
Adding sectoring:	*capacity*	increases	decreases	no change
Larger path loss exponent (β):	*capacity*	increases	decreases	no change

(circle correct answer)

102. A system operates with a power margin of 6 dB for outage due to thermal noise at the cell edge. If the maximum outage probability at the cell edge is 0.01, what is the maximum shadowing deviation that can be tolerated?

103. **True or False:** The pulse shape defined by

$$p(t) = \begin{cases} e^{-t/T} & 0 \le t < T \\ 0 & \text{otherwise} \end{cases}$$

satisfies the Nyquist criterion.

104. Consider a system with reuse factor $N_c > 1$. Holding all other parameters fixed, indicate how the SIR due to CCI (worst case forward link) varies with:

Increasing reuse factor (N_c):	SIR	increases	decreases	no change
Adding sectoring:	SIR	increases	decreases	no change
Decrease cell radius:	SIR	increases	decreases	no change
Larger path loss exponent (β):	SIR	increases	decreases	no change

(circle correct answer)

105. **True or False:** In a direct sequence system, if the delay spread of the channel is several chip times, the channel is frequency-selective.

106. Time diversity in a mobile communication system is typically obtained by coding and

    (a) equalization

    (b) interleaving

    (c) frequency hopping

107. **True or False:** It is better to have two orders of spatial diversity than two orders of frequency diversity because a better error performance is achieved.

108. The average power of the short-term fading model is set by the

    (a) System operator

    (b) Parameters defined at run-time

    (c) Path-loss model and instantaneous shadowing realization

109. One strength of CDMA-based systems relative to FDMA/TDMA designs is that more frequency diversity is typically achieved and

    (a) Combined using no channel estimation

    (b) Combined relatively easily, naturally yielding soft-decision information

    (c) Combined using a Viterbi-based equalizer

110. **True or False:** For an unspread system in a fading channel, Intersymbol Interference yields diversity against fading.